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Time series modelling of monthly reference evapotranspiration for Bikaner, Rajasthan (India)

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ABSTRACT

Meteorological data were collected from Indian Meteorological Department (Pune) for Bikaner district of Rajasthan from the year 1961 to 2005 and monthly reference Evapotranspiration (ET_0) was estimated using the Penman-Monteith FAO-56 method. For smoothening data and to stablise the variance in the data series of monthly $ET_{0,}$ cube root transformation was applied. Monthly ET_0 (cube root transformation) data from the year 1961 to 2000 were taken for time series modelling and remaining data from the year 2001 to 2005 were used for model validation. Turning point and Mann-Kendall tests were used at 5% significant level for identifying trend component. A trend- free monthly ET_0 series were used for modelling the periodic component using Fourier series analysis. First 12 harmonics explained total variance of 173.47% for monthly (cube root transformation) ET_0 series. Hence, all 12 harmonics were considered. Before modelling stochastic dependent component, periodic component was removed from the time series and series was made stationary. For modelling dependent stochastic component autoregressive (AR) / moving average (MA) / autoregressive moving average (ARMA) / autoregressive integrated moving average (ARIMA) models were tried. ARIMA (12, 1, 1) model was fond the best fit model based on the minimum value BIC statistics. The dependent stochastic component was separated from the series to obtain new series (a_t) of independent stochastic component. Portmanteau test and Box-Cox transformation was applied to series a_t for checking independence and normalization, respectively. Time series models were developed by adding deterministic (trend and periodic) and stochastic (dependent and independent) components. Model was evaluated with regards to several statistical measures. The correlation coefficient and Nash-sutcliffe coefficient also indicated high degree of models fitness to the observed data. Developed time series model was validated with 5 years values of monthly ET_0 (cube root transformation). Using the developed time series model, monthly ET_0 were forecasted for the year 2006 to 2050.

1. INTRODUCTION

The science of evapotranspiration has been advanced greatly during last many years, and is still evolving. Recognizing that evapotranspiration is one of the basic component of the hydrologic cycle, the Food and Agriculture Organization (FAO) adopted the concept of reference evapotranspiration (ET_0) in the FAO guidelines for crop water requirements based on studies by Doorenbos and Pruitt (1975 and 1977), which are widely accepted for calculating evapotranspiration. Reference evapotranspiration is based primarily on measurements of radiation, vapour pressure, air temperature, relative humidity, and wind velocity over the evaporating surface. Many investigators have developed equations to estimate evapotranspiration. The commonly used equation to estimate reference evapotranspiration is the Penman-Monteith FAO-56 (Kale *et al.,* 2013; Bandyopadhyay *et al.,* 2009, Jhajharia *et al*., 2015; Panigrahi *et al.,* 1992 and Panigrahi *et al.,* 2002). This method includes both physiological and aerodynamic parameters (Xu *et al.,* 2006), and is the most reliable and universally accepted method to estimate evapotranspiration under various types of climate.

Jhajharia *et al*. (2009) carried out a study to assess reference evapotranspiration by temperature-based methods for humid regions of Assam. Blaney Criddle, Hargreaves and Thronthwaite methods were used to estimate ET_0 for the study area using monthly meteorological data. The FAO Penman-Monteith method was selected as the standard method for comparison to evaluate above three temperature based methods. Meshram *et al.* (2010) carried out a study on reference crop evapotranspiration of western part of Maharahtra, India. Six reference crop evapotranspiration (ET_0) methods were studied based on their daily performances under the given climatic condition in the study area. The Penman-Monteith equation standardized by Food and Agricultural Organization (FAO-56-PM) was used to compare with the Modified Penman, Hargreaves-Samani, Pan Evaporation, Blaney Criddle and FAO Radiation methods using meteorological data for 33 years (1975-2007). George and Raghuwanshi (2012) used six empirical methods for calculating ET_0. The ET_0 values estimated by all six methods were compared with the FAO-56 Penman-Monteith ET_0 estimates, which were taken as the standard. Chauhan and Shrivastava (2012) carried out a research on estimating reference evapotranpiration using neural computing technique with climatic data required for Penman-Monteith method and compared the performance of ANNs with Penman-Monteith method. Paudel and Pandey (2013) carried out a study in which five ET_0 methods were applied for the water balance estimation of Sikta irrigation command area.

The importance of time series analysis has become a major tool in engineering hydrology. It is being used either for building mathematical model to generate synthetic hydrologic records or to forecast the occurrence hydrology event ahead of time (Dabral *et al*., 2014). Jadhav and Bhakar (2009) had developed the additive type time series model of gram evapotranspiration using 26 years (1978-2003) data of Udaipur (India). Out of 26 years data, 24 years data were used for model development and remaining two years data used for validation. The trend tests indicated that the evapotranspiration series was trend free. The periodic component was represented by second harmonic expression. The stochastic components of the evapotranspiration follow third order AR model. Bhakar *et al*. (2006), Dabral *et al.* (2008, 2014 and 2016) developed time series model for different hydrological parameters. Since limited studies are available on time series modelling of reference evapotranspiration (ET_0) for semi-arid region of India, the present study was under taken to develop and validate the time series model for monthly evapotranspiration for Bikaner Rajasthan) and future prediction.

2. MATERIALS AND METHODS

Study Area

The present study area Bikaner ($71^0$54' to $74^0$12'E and $27^0$11' to $29^0$3'N with Altitude 237 m above mean sea level) is situated in the middle of Thar desert and has a hot-arid climate with low rainfall and extreme temperature. During summer season it very hot (28 to 53^0C) and in winter very cold (-4^0C to 23.2^0C), Annual rainfall is in the range of 260-440 mm. The different meteorological data required for the computation of ET_0 are maximum temperature (^{0}C), minimum temperature (^{0}C), relative humidity (%), wind speed (km h^{-1}) and duration of actual bright sunshine (h). The monthly data of these climatic parameters were obtained from the Indian Meteorological Department (Pune), Maharashtra, for the station of Bikaner from 1961 to 2005. ET_0 was estimated using Penman-Monteith FAO-56 method. For estimating ET_0 procedure referred by Allen *et al*. (1998) was adopted.

Time Series and Modelling of Deterministic and Dependent Stochastic Component

The additive form of time series model is expressed as:

$$X_t = T_t + P_t + S_t + a_t \quad ...(1)$$

Where, T_t = Deterministic trend component, P_t = Deterministic periodic component, S_t = Stationary stochastic dependent component and a_t = Stationary stochastic independent component, t = 1, 2, 3 …… N, where N = total number of values and X_t = Monthly ET_0 data.

In this study, the data series was transformed by using cube root transformation. This was done for smoothening and stabilising the variance in the series. Firstly trend and periodic components were identified and removed from original series using the methodology as described in the works of Dabral *et al*. (2008 and 2016) and Jhajharia *et al*. (2015). The left series were consist of stochastic component. It was standaridesed and series were made stationary before doing further stochastic modelling. In this study auto-regressive (AR), moving average (MA), auto-regressive moving average (ARMA) and auto-regressive integrated moving average (ARIMA) models were tried. The model parameters were estimated using SPSS-16 software. The autocorrelation and partial autocorrelation functions were also computed for the series S_t along with their standard error and plotted against lag *K*. BIC statistics were also estimated. The order of the models was identified on the basis of the minimum value of BIC statistics.

Modelling of Independent Stochastic Component

The dependent stochastic component was separated from the series. The new series (a_t) was called as independent stochastic component. To check the independence of a_t, the Portmanteau test formulated by McLeod and Li (1983) was carried out. It was found that the residual series a_t was having some negative values, therefore, to convert all values of series, a_t, into positive values, an addition of 2 was made in residual series of monthly (cube root transformation) ET_0. The coefficient of skewness of series, a_t was found to be quite different from zero, therefore, series a_t is not normally distributed. So, for normalisation Box-Cox transformation was done. The transformation is mathematically expressed as:

$$y = \frac{X^{\lambda} - 1}{\lambda} \quad ; \lambda \neq 0 \qquad \text{...(2)}$$

$$y = \ln x \quad ; \lambda = 0.5$$

Where, y is the transformed value of the variate x in the given series a_t, the parameter λ was selected such that the skewness of the transformed data series is minimum. Hinkley (1977) suggested a simpler procedure for the transformation, According to the test statistic d_λ is calculated for each of the different value of λ varying from -1 to +1 with an interval of 0.1 and coefficient of skewness were computed. The test statistic d_λ is expressed as:

$$d_\lambda = \frac{\bar{y} - \varepsilon_y}{\sigma_y} \qquad \text{...(3)}$$

Where, $\bar{y}$ = mean of the transformed series, ε_y = median of the transformed series, and σ_t = standard deviation of transformed series.

The value of λ for which d_λ is minimum was used for the transformation into a normal series. For modelling the independent stochastic component (a_t), normal probability distribution function was fitted to a_t series as suggested by Chow *et al.* (1988). The transformed R_t series, in terms of a_t series, can be represented as:

$$R_t = \mu_t + \sigma_t z_t \qquad \text{...(4)}$$

Where, R_t = transformed a_t series, μ_t = mean of the R_t series, σ_t = standard deviation of the R_t series and z_t = a random component with zero mean and unit variance (available from standard table).

Evaluation of Regeneration Performance

The models were evaluated with regard to several statistical measures such as absolute error, relative error, nash-sutcliffe coefficient (E_{NS}), integral square error, mean relative error, root mean square error and mean forecasted error.

3. RESULTS AND DISCUSSION

Monthly ET_0 Variation using Penman-Monteith FAO-56

The monthly ET_0 values obtained using the Penman-Monteith FAO-56 method from 1961-2005

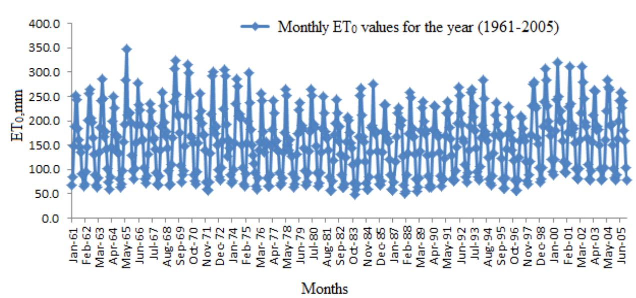

Fig. 1. Estimated value of monthly ET_0 for the year 1961 to 2005

for Bikaner is shown in Fig. 1. The estimated monthly ET_0 value ranges from 49.0 mm to 347.5 mm for the year 1961-2005. The average monthly ET_0 was found to be 155.5 mm. The monthly ET_0 values were found to be low in the months of January, February, October, November and December and high during the months of March, April, May, June, July, August and September.

Deterministic Modelling of ET_0

For identifying the trend component in the monthly (cube root transformation) ET_0 series, firstly the turning point test was carried out. The hypothesis of trend in the series was formulated and checked. The value of the p, the E (p) and the Var (p) are given in Table 1. Results indicated the presence of the trend component in the monthly (cube root transformation) ET_0 series because the calculated value of Z obtained through the turning point test was found outside the critical range of ±1.96 at the 5% level of significance. Thus the hypothesis of no trend was rejected.

Further, non-parametric Mann Kendall's test was also used to identify the trend in the monthly (cube root transformation) ET_0 series. The value of Z statistics has been found to be outside the critical range of ±1.96 at the 5% level of significance in monthly (cube root transformation) ET_0 series and thus indicated the presence of a trend (Table 1).

Comparing the overall statistics of turning point and Man Kendall's tests, the observed monthly (cube root transformation) ET_0 series data could not be considered to be trend free. Using regression analysis, equations of trend (T_t) were determined and given in Table 1. After removing the trend, a trend free series was obtained for a monthly (cube root transformation) ET_0.

Autocorrelogram of the series Y_t was developed for identification of the base period. For the development of Autocorrelogram, the auto correlation function r_k of series Y_t for lag1 to 200 for monthly (cube root transformation) ET_0 was calculated and is shown in Fig. 2. The auto correlogram of series Y_t for monthly ET_0 (cube root transformation) shows that autocorrelation functions are significantly different from zero, which indicates that the monthly (cube root transformation) values of Y_t series are mutually dependent. The peak and trough of the auto-correlogram show that the series Y_t for monthly ET_0 (cube root transformation) has a periodic component with a base period of 12 months.

Fourier decomposition of periodic component and cumulative periodogram of the monthly (cube root transformation) ET_0 series of Y_t at Bikaner is shown in Table 2 and Fig. 3, respectively. All the 12 harmonics for the monthly ET_0 explained a total variance of 173.47% for Bikaner station (Table 2 and

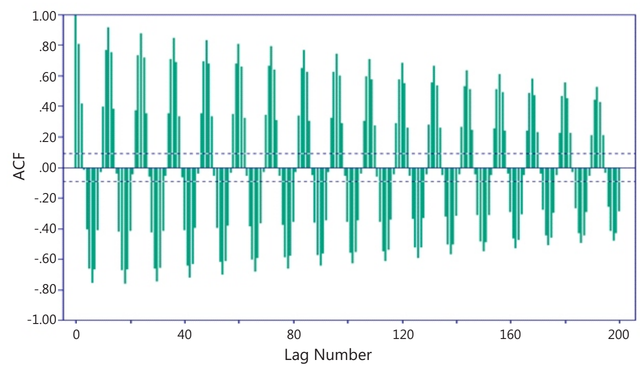

Fig. 2. Autocorrelogram of series Y_t

Table: 2
Fourier decomposition of periodic component of monthly ET_0 (Yt) at Bikaner

Order	A_j	B_j	Explained variance (%)	Cumulative explained variance (%)
1	0.35	-0.06	10.16	10.16
2	0.15	0.06	2.18	12.34
3	0.12	0.03	1.28	13.63
4	-0.77	-0.68	84.94	98.57
5	0.13	0.66	36.39	134.96
6	0.07	0.13	1.88	136.84
7	-0.004	-0.007	0.01	136.84
8	0.26	-0.46	22.20	159.04
9	0.14	-0.18	4.08	163.13
10	-0.12	-0.02	1.14	164.27
11	0.09	-0.09	1.37	165.64
12	0.24	-0.21	7.83	173.47

Table: 1
Analysis of trend component for monthly ET_0 (Turning point test, Mann-Kendal's test) and equation of trend line

Variable	Transformation	Turning point test					Mann Kendal's test			Equation of Trend line
		N	P	E(P)	Var (P)	Z	S	Variance of S	Z	
Monthly ET_0	Cube Root	480	125	318.67	85.01	-21.005	-1381	476100	-2.00	T_t=-0.0001x+5.2661

Fig. 3). Therefore, all twelve harmonics were considered for modelling. Periodic means and periodic standard deviation were computed. The periodicity of the series Y_t was removed. The remaining S_t Series was subjected to checks for stationary behaviour and model identification.

Modelling of Dependent Stochastic Component

For monthly ET_0 series (S_t), it is observed that the autocorrelation and partial auto correlation functions at lag 1 to 12 crossed the 95% confidence limit and the values of autocorrelation and partial auto correlation functions is diminishing with the increase of lag numbers (Fig. 4 and Fig. 5). This indicates that AR, MA, ARMA and ARIMA models may be tried for monthly ET_0 series (S_t). For monthly ET_0 series (S_t), ARIMA (12, 1, 1) model was found the best fit based on the minimum value of BIC statistics (Table 3).

Modelling of Independent Stochastic Component

The calculated value of statistics of Portmanteau test formulated by Ljung and Box (1978) was found to be 72 for a_t series of monthly ET_0 (cube root transformation) which is less than the tabular value of 76.15 at 1% level of significance at 50 degrees of freedom.This indicated that a_t series of independent stochastic component for monthly (cube root transformation) ET_0 consists of independent and identically distributed variables. Before modelling the independent stochastic component, the normality of series a_t was checked by finding the values of the coefficient of skewness, since a normal distribution has a coefficient of skewness equal to zero. It was found that the residual series a_t was having some negative values, therefore, to convert all values of the series, a_t, into positive values, an addition of 2 was made in a_t residual series of monthly (cube root transformation) ET_0. The coefficient of skewness was found to be -0.48115 in a_t residual series of monthly ET_0 which is quite different from zero, therefore, series a_t is not a normally distributed series. To obtain a normalized form of series a_t, Box- Cox transformation was applied as discussed in section 2.8. According to that test d_λ, is calculated for each of different value of λ varying from -1 to +1 with an interval of 0.1 and coefficient of skewness was computed.

The value of statistics d_λ and the coefficient of skewness were found to be minimum for λ = 0.9 and the values were 0.0097 and -0.48115, respectively.

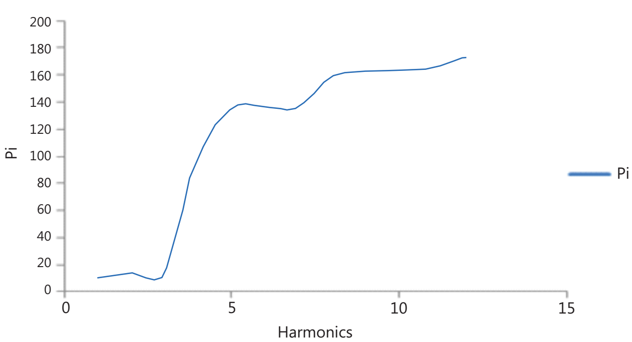

Fig. 3. Cumulative periodogram of monthly ET_0 at Bikaner

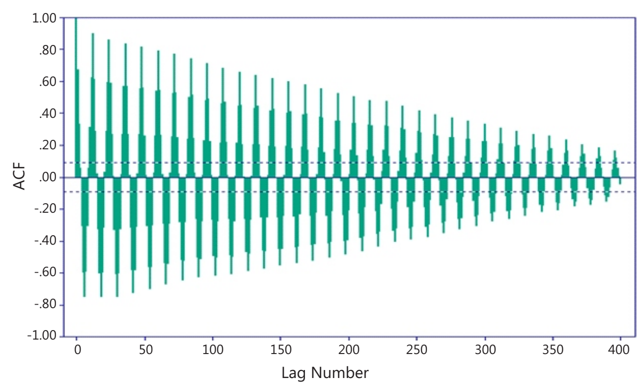

Fig. 4. Autocorrelogram of series St, monthly ET_0 with S.E. limit

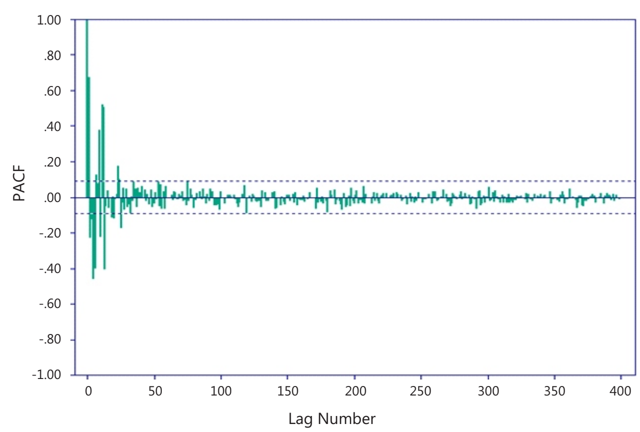

Fig. 5. Partial Autocorrelogram of series St, monthly ET_0 with S.E. limit

Table: 3
The best fit autoregressive models for series S_t

Variable	Transformation	Best fit model	Description of model	BIC
Monthly ET_0	Cube root	ARIMA (12,1,1)	S_t=0.889$_{St-1}$-0.19$_{St-2}$+0.216$_{St-3}$-0.207$_{St-4}$+0.159$_{St-5}$-0.214$_{St-6}$+0.202$_{St-7}$-0.151$_{St-8}$ +0.218$_{St-9}$-0.211$_{St-10}$+0.18$_{St-11}$+0.698$_{St-12}$-0.589$_{St-13}$+R_t-0.338R_{t-1}	-2.255

Therefore, suitable value of λ for Box-Cox transformation was obtained 0.9. For modelling the independent stochastic component, normal probability distribution function was fitted to monthly ET_0 series (cube root transformation) (a_t) obtained after the Box-Cox transformation. Calculated Smirnov-Kolmorgorovs's statistics value was obtained 0.0613 which is less than that of tabular value 0.0621 at the 5% level of significance. After substituting transformed residual series a_t in equation, the series was expressed as:

$$R_t = \mu_t + \sigma_t z_t$$

Putting the value of R_t in the above equation,

Where, R_t = transformed a_t series, μ_t = mean of the series R_t, σ_t = standard deviation of series R_t and z_t = a random component with zero mean and unit variance. The mean (μ_t) and standard deviation (σ_t) of the series a_t were found to be 0.96 and 0.275, respectively.

Time Series Model

Time series models for monthly (cube root transformation) ET_0 was developed by substitution of the values of deterministic and stochastic Components in eq. 2. The decomposition models of the time series (X_t) is given in Table 4.

The qualitative assessment was made by the comparison of autocorrelation functions of the historical and generated series for monthly ET_0. The autocorrelogram upto 400 lags of historical and generated time series for monthly ET_0 is presented in Fig. 6 and Fig. 7. It is seen from the figures that the properties propagated in the generated series are similar to the historical series.

Model Assessment

The results of the time series model were evaluated quantitatively to assess the regeneration

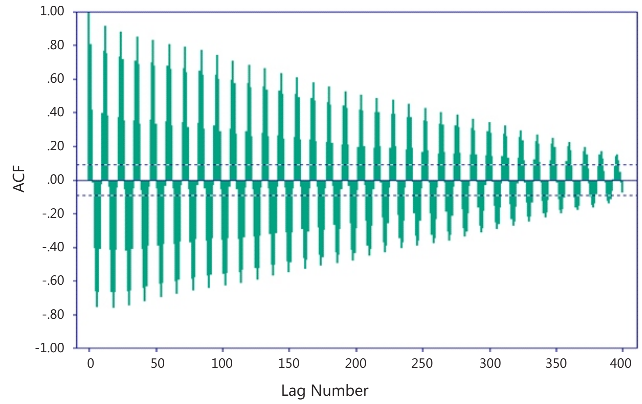

Fig. 6. Autocorrelogram of actual series, X_t

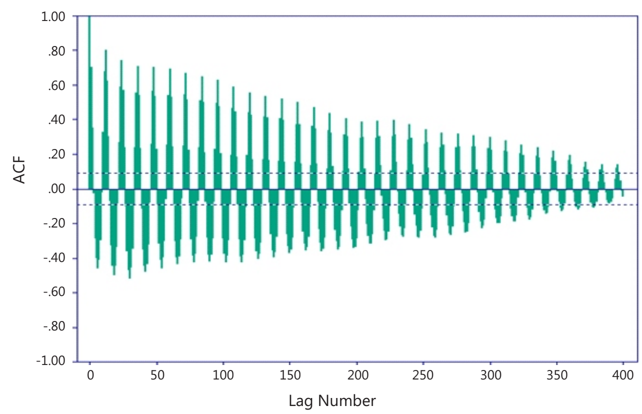

Fig. 7. Autocorrelogram of generated series, X_t

Table: 4
Decomposition of time series models

Variable	Transformation	Decomposition of model
Monthly ET_0	Cube root	$0.0001 \times 5.266 - 0.001736781 + (0.350227964 \cos\left(\frac{2\pi}{12}\right)\tau + (-0.06203) \sin(\frac{2\pi}{12})\tau + (0.152540579) \cos(\frac{4\pi}{12})\tau + (0.063254) \sin\frac{4\pi}{12}\tau + (0.123053028) \cos\frac{6\pi}{12}\tau + (0.028703) \sin\frac{6\pi}{12}\tau + (-0.768242393) \cos\frac{8\pi}{12}\tau + (-0.68409) \sin\frac{8\pi}{12}\tau + (0.1268278) \cos\frac{10\pi}{12}\tau + (0.661246) \sin\frac{10\pi}{12}\tau + (0.073040371) \cos\frac{12\pi}{12}\tau + (0.13426) \sin\frac{12\pi}{12}\tau + (-0.004264952) \cos\frac{14\pi}{12}\tau + (-0.00676) \sin\frac{14\pi}{12}\tau + (0.260183891) \cos\frac{16\pi}{12}\tau + (-0.45702) \sin\frac{16\pi}{12}\tau + (0.138148281) \cos\frac{18\pi}{12}\tau + (-0.17833) \sin\frac{18\pi}{12}\tau + (-0.116908588) \cos\frac{20\pi}{12}\tau + (-0.0243) \sin\frac{22\pi}{12}\tau + (0.235466944) \cos\frac{24\pi}{12}\tau + (-0.20516) \sin\frac{24\pi}{12}\tau + (0.889S_{t-1} - 0.19S_{t-2} + 0.216S_{t-3} - 0.207S_{t-4} + 0.159S_{t-5} - 0.214S_{t-6} + 0.202S_{t-7} - 0.151S_{t-8} + 0.218S_{t-9} - 0.211S_{t-10} + 0.18S_{t-11} + 0.698S_{t-12} - 0.589S_{t-13} + R_t - 0.338R_{t-1}\left(((0.8644725 + 0.2481138 \times z_t) + 1)^{\frac{10}{9}}\right) - 2)$

performance. The model was evaluated with regards to several statistical measures such as mean, median, standard deviation, the coefficient of skewness, correlation coefficient and other statistical parameters.

The mean, median, standard deviation and the coefficient of skewness of the generated series (1961-2000) were found 5.10 mm, 5.18 mm, 0.67 mm and -0.11 which were close to the mean of 5.24 mm, median of 5.38 mm, standard deviation of 0.79 mm and the skewness coefficient of -0.11 of the historical series (Table 5). The correlation and nash-sutcliffe coefficients were obtained 0.902 and 0.997, respectively, indicating a high degree of model fitness (Table 6).

Mean monthly values of ET_0 from 1961 to 2000 of historical data and generated sequence were used for comparison. The respective relative, absolute errors, integral square error, mean relative error, mean forecasted error and root mean square error of all the calendar months were calculated for comparison and are given in Table 6. The absolute error was found in the range of 0.01 mm to 0.48 mm. It was found low in the months of January, February, October, November and December. Relative error was found in the range of 0.13 to 7.66. Relative error was found to be low in the months January, February, October and December. The integral square error was found in the range of 0.006 to 0.016. The integral square error was found to be low in the months January, February, March, April, October, November and December. The mean relative error was found in the range of 0.00003 to 0.002. The mean relative error was found to be low in the months January, February, March, October, November and December. The mean forecasted error was found in the range of -0.143 mm to 0.482 mm. The mean forecasted error was found to be low in the months of January, February, March, October, November and December. The root mean square error was found in the range of 0.16 mm to 0.64 mm. The root mean square error was also found to be low in the months of January, February, March, October, November and December. All the values of the errors indicate good model fitness.

Fig. 8 shows 1:1 line criteria which clearly shows that model almost equally overestimate and underestimate ET_0 values. It seems to be fairly fit model.

Validation of the Developed Time Series Models

The developed time series model was used for prediction of 5 years values of monthly ET_0 *i.e.* for the years 2001 to 2005. The predicted values of monthly

Table: 5
Statistical parameters of monthly ET_0 (cube root transformation) for the year 1961-2000

Variable	Statistical parameters	Mean (mm)	Median (mm)	Standard deviation (mm)	Coefficient of skewness
Monthly ET_0	Historical values	5.24	5.38	0.79	-0.11
(cube root transformation)	Predicted values	5.10	5.18	0.67	-0.11

Table: 6
Mean monthly ET_0 (cube root transformation) of historical and generated data series (1961-2000) along with errors

Months	Mean of historical data (mm)	Mean of generated data(mm)	Errors		Integral square error	Mean relative error	MFE (mm)	RMSE (mm)	Correlation coefficent	Nash sutcliff coefficient
			Absolute error (mm)	Relative error (%)						
Jan	4.10	4.15	0.04	1.09	0.0083	0.00027	-0.045	0.22	0.902	0.997
Feb	4.46	4.48	0.02	0.49	0.0075	0.00012	-0.022	0.21		
Mar	5.22	5.34	0.11	2.16	0.0069	0.00054	-0.113	0.23		
Apr	5.75	5.54	0.21	3.59	0.0099	0.00090	0.206	0.36		
May	6.33	5.97	0.36	5.66	0.0113	0.00142	0.358	0.45		
June	6.30	5.82	0.48	7.66	0.0160	0.00192	0.482	0.64		
July	5.80	5.38	0.43	7.34	0.0146	0.00183	0.426	0.54		
Aug	5.59	5.36	0.24	4.25	0.0118	0.00106	0.238	0.42		
Sep	5.54	5.26	0.29	5.16	0.0114	0.00129	0.286	0.40		
Oct	5.18	5.19	0.01	0.13	0.0066	0.00003	-0.007	0.21		
Nov	4.49	4.64	0.14	3.18	0.0088	0.00079	-0.143	0.25		
Dec	4.11	4.13	0.03	0.66	0.0060	0.00016	-0.027	0.16		

ET_0 along with the historical values for the year 2001 and 2005 is shown in Fig. 8. The mean, median, standard deviation and the coefficient of skewness of the generated series (2001 to 2005) are found to be 5.35 mm, 5.26 mm, 0.82 mm and 0.25 which is close to the mean of 5.39 mm, median of 5.45 mm, standard deviation of 0.76 mm and the coefficient of skewness of -0.01 of the historical series (Table 7). The correlation and Nash-Sutcliffe coefficients were obtained 0.81 and 0.60, respectively, indicating a good degree of model fitness (Table 8).

Mean monthly values of ET_0 from 2001 to 2005 of historical data and generated sequence were used for comparison. The respective relative, absolute errors, integral square error, mean relative error, mean forecasted error and root mean square error of all the calendar months were calculated for comparison and are given in Table 8. The absolute error was found in the range of 0.03 mm to 0.84 mm. The absolute error was found to be low in the months of February, July and November. Relative error (%) was found in the range of 0.49 to 15.55. Relative error was found to below in the months of February, July and November. The integral square error was found in the range of 0.02 to 0.07. The integral square error was found to be low in the months of June and July. The mean relative error was found in the range of 0.01 to 0.021. The mean relative error was found to be low in the months of February, July and November. The mean forecasted error was found in the range of -0.41mm to 0.84 mm. The mean forecasted error was found to be low in the months of January, February, March, April, June, July and December. The root mean square error was found to be in the range of 0.25 mm to 0.84 mm. The root mean square error was found to be low in the months of February, June, July and December. All the values of the errors indicate good model fitness.

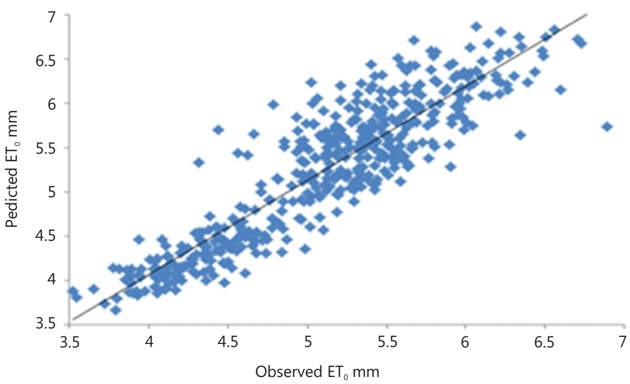

Fig. 8. Observed and predicted values of monthly ET_0 for the year (1961-2000)

Fig. 9 shows 1:1 line criteria which indicate that model during validation, underestimate little more

Table: 7
Statistical parameters of monthly ET_0 (Cube root transformation) for the year 2001-2005

Variable	Statistical parameters	Mean (mm)	Median (mm)	Standard deviation (mm)	Coefficient of skewness
Monthly ET_0	Historical values	5.39	5.45	0.76	-0.01
(cube root transformation)	Predicted values	5.35	5.26	0.82	0.25

Table: 8
Mean monthly ET_0 (cube root transformation) of historical and generated data series (2001-2005) alongwith errors

Months	Mean of historical data (mm)	Mean of generated data(mm)	Errors		Integral square error	Mean relative error	MFE (mm)	RMSE (mm)	Correlation coefficent	Nash sutcliff coefficient
			Absolute error (mm)	Relative error (%)						
Jan	4.35	4.65	0.31	7.02	0.04	0.014	-0.31	0.41	0.81	0.60
Feb	4.65	4.72	0.07	1.47	0.04	0.003	-0.07	0.37		
Mar	5.46	5.88	0.41	7.59	0.04	0.015	-0.41	0.53		
Apr	5.99	6.17	0.18	3.00	0.03	0.006	-0.18	0.41		
May	6.55	6.34	0.21	3.20	0.03	0.006	0.21	0.43		
June	6.39	6.59	0.20	3.16	0.02	0.006	-0.20	0.25		
July	5.60	5.63	0.03	0.49	0.02	0.001	-0.03	0.28		
Aug	5.86	5.25	0.61	10.48	0.05	0.021	0.61	0.67		
Sep	5.44	5.28	0.16	2.85	0.04	0.006	0.16	0.45		
Oct	5.39	4.55	0.84	15.55	0.07	0.031	0.84	0.84		
Nov	4.69	4.61	0.09	1.85	0.05	0.004	0.09	0.48		
Dec	4.31	4.59	0.27	6.38	0.04	0.013	-0.27	0.37		

ET_0 values compared to overestimate values. But based on the different error estimate, model seems to be well validated.

Forecasting of Monthly ET_0

Using the developed time series models (Table 5), forecasted values of monthly ET_0 for the year 2006 to 2050 are presented in Fig. 10. The forecasted monthly ET_0 ranges from 40.9 mm to 468 mm. The average value of monthly ET_0 was found 157.3 mm. The Forecasted values of monthly ET_0 may be useful for water management planning of the study area.

4. CONCLUSIONS

The monthly estimated ET_0 values obtained using the Penman-Monteith FAO-56 ranged from 49.0 mm to 347.5 mm for Bikaner. The average monthly ET_0 was found to be 155.5 mm. The monthly ET_0 values were found low in the months of January, February, October, November and December and higher during the months of March, April, May, June, July, August and September. Monthly reference evapotranspiration (cube root transformation) series was having deterministic (trend and periodic) and stochastic (dependent and independent) components. Model was evaluated with regards to several statistical measures. Nash-sutcliffe coefficient (0.977) indicated high degree of model fitness to the observed data.

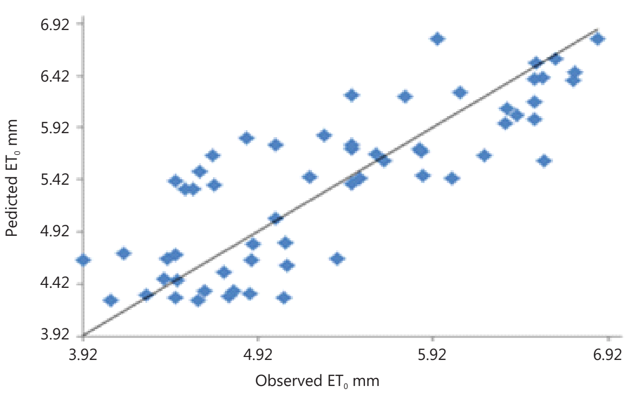

Fig. 9. Observed and predicted values of monthly ET_0 for the year (2001-2005)

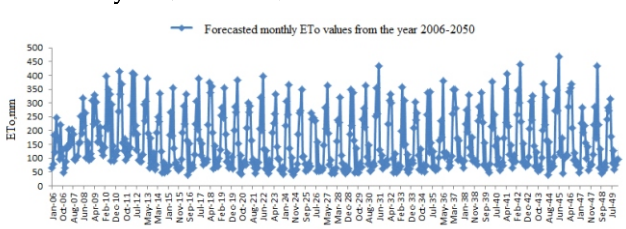

Fig. 10. Forecasted value of monthly ET_0 (2006-2050)

Validation of the developed time series model for monthly reference evapotranspiration (cube root transformation) was done using 5 years data (2001-2005). The Nash-sutcliffe coefficient (0.60) also indicated a good degree of model fitness to the observed data. Monthly reference evapotranspiration was forecasted for the year 2006 to 2050 using the developed time series model.

REFERENCES

Allen, R.G., Preeria, L.S., Raes, D. and Smith, M. 1998. Crop evapotranspiration guidelines for computing crop requirements. FAO Irrigation Drainage Paper No. 56, FAO: Rome, 300 p.

Bandyopadhyay, A., Bhadra, A., Raghuwanshi, N.S. and Singh, R. 2009. Temporal trends in estimates of reference evapotranspiration over India. *J. Hydrol. Eng.*, 14: 508-515.

Bhakar, S.R., Raj Vir Singh and Hari Ram 2006. Developed stochastic modelling of wind using 26 years (1978-2003) data. *J. Agric. Eng.*, 43(1): 1-7.

Chauhan, S. and Shrivastava, R.K. 2012. Estimating reference evapotranspiration using neural computing technique. *J. Ind. Water Resour. Soc.*, 32: 1-2.

Chow, V.T, Maidment, D.R. and Mays, L.W. 1988. Applied Hydrology. McGraw-Hill, New York.

Dabral, P.P., Pandey, A., Baithuri, N. and Mal, B.C. 2008. Stochastic modelling of rainfall in humid region of north east India. *Water Resour. Manage.*, 22: 1395-1407.

Dabral, P.P., Jhajharia, D., Mishra, P., Hanshing, L. and Dolley, J. 2014.Time series modelling of pan evaporation: A case study of the northeast India. *Global Nest J.*, 20(10): 281-293.

Dabral, P.P., Saring, T. and Jhajharia, D. 2016. Time series models of monthly rainfall and temperature to detect climate change for Jorhat (Assam), India. *Global NEST J.*, 18(3): 494-507.

Doorenbos, J. and Pruitt, W.O. 1975. Guidelines for predicting crop water requirements. *FAO Irrigation Drainage Paper No. 24*. FAO: Rome, 168 p.

Doorenbos, J. and Pruitt, W.O. 1977. Crop water requirements. *FAO Irrigation Drainage Paper No. 24*. FAO: Rome, 144 p.

George, B.A. and Raghuwanshi, N.S. 2012. Intercomparison of reference evapotranspiration estimated using six methods with data from four climatological stations in India. *J. Ind. Water Resour. Soc.*, 32: 3-4.

Hinkley, D. 1977. On quick choice of power transformation. *Appl. Statistics*, 26: 67-69.

Jadhav, K.A. and Bhakar, S.R. 2009. Stochastic modelling of gram evapotranspiration under the climatic conditions of Udaipur. *J. Ind. Water Resour. Soc.*, 29(1): 1-7.

Jhajharia, D., Ali, M.I., Debarma, S., Durbude, D.G. and Kumar, R. 2009. Assessing evapotranspiration by temperature-based methods for humid regions of Assam. *J. Indian Water Resour. Soc.*, 29(2): 1-8.

Jhajharia, D., Kumar, R., Dabral, P.P., Singh, V.P., Choudhary, R.R. and Dinpashoh, Y. 2015. Reference evapotranspiration under changing climate over the Thar desert in India. *Meteorol. Appl.*, 22(3): 425-435.

Kale, M.U., Nagdeve, M.B. and Bagade, S.J. 2013. Estimation of evapotranspiration with ANN technique. *J. Ind. Water Resour. Soc.*, 33(1): 23-29.

Ljung, G.M. and Box, G.E.P. 1978. On a measure of lack of fit in time series models. *Biometrika,* 65(2): 297-303.

McLeod, A.I. and Li, W.K. 1983. Diagnostic checking ARMA time series models using squaredresidual autocorrelations. *J. Time Ser. Anal.*, 4(4): 269-273.

Meshram, D.T., Mittal, H.K., Gorantiwar, S.D. and Purohit, R.C. 2010. Reference crop evapotranspiration of Western part of Maharashtra, India. *Indian J. Soil Cons.*, 38(2):80-85.

Panigrahi, B., Sharma, S.D. and Behra, B.P. 1992. Irrigation water requirement models of some major crops. *Water Resour. Manage.*, 6(1): 69-77.

Panigrahi, B., Panda, S.N. and Mull, R. 2002. Prediction of hydrological events for planning rainfed rice. *Hydrol. Sci. J.*, 47(3): 435-448.

Paudel, H.D. and Pandey, A. 2013. Comparative study of reference evapotranspiration estimation methods for the water balance estimation of Sikta irrigation project, Nepal. *J. Indian Water Resour. Soc.*, 33: 4.

Xu, Chong-Yu, Gong, Lebing, Jiang, Tong, Chen, Deliang and Singh, V.P. 2006. Analysis of spatial distribution and temporal trend of reference evapotranspiration and pan evaporation in Changjiang (Yangtze River) catchment. *J. Hydrol.*, 327: 81-93.