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Development of additive form of time series models for rainfall

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ABSTRACT

Using monthly, weekly and monsoon rainfall data (1988 to 2016), additive form of time series models were developed. Turning point and Mann-Kendall (M-K) tests were carried out for identifying the trend component and Fourier series analysis were used for modelling the periodic component. Modelling of dependent stochastic component was done using ARMA (4,0,8) model for monthly rainfall and AR (1) model for both weekly and daily monsoon rainfall. The Portmanteau test was used to check the independence of stationary stochastic independent component. Using Box-Cox transformation, independent stochastic component was normalised and its modelling was done using normal distribution function. Time series models were evaluated using several statistical measures, and they indicated a good model fitness. Developed time series models were also validated with 8 years rainfall data.

1. INTRODUCTION

Hydrological modelling using climatological data as a stochastic process is used in a variety of hydrological areas. A stochastic model is developed to generate alternative hydrological data sequences that are likely to occur in future based on characteristics of past historical records. Rainfall modelling is an important area of hydrology in which research is still being actively carried out. Long sequences of rainfall data are an essential component for a better decision and proper planning of land and water management projects (Verma and Kumar, 2012).

A time series model has two components - deterministic and stochastic. Deterministic component are periodic or non-periodic. The non-periodic component normally has trend and jump characteristics. Trend may be an increasing type or a decreasing type. The periodic component has cyclic pattern at fixed interval. A stochastic component may have irregular oscillation and random effects (Das, 2009). The importance of time series analysis has gained great aspects in engineering hydrology. Reddy and Kumar (1999) carried out the time series study for monthly rainfall for Bino watershed of Ramganga river (India). Raja Kumar and Kumar (2004) used AR(2) and ARMA (2, 2) models based on the goodness of fit tests for daily rainfall of south-west monsoon season of Baptala, Andhra Pradesh (India). Raja Kumar and Kumar (2007) developed a time series model of daily rainfall during north-east monsoon season of Baptala, Andhra Pradesh (India). They found the performance of ARMA (2, 2) model better than AR(2) model. Sherring *et al.* (2009) used AR (1) model for prediction of rainfall in Lider catchment of South Kashmir. Zakaria (2011) studied stochastic characteristics of daily rainfall series of Purajaya region. He observed daily rainfall data trend free; the periodic component was represented by 253 harmonics and stochastic components of daily rainfall series was modelled using AR(2). Verma and Kumar (2012) developed a stochastic model of monthly rainfall for Jodhpur. Meher and Jha (2013) developed ARIMA model for simulating and forecasting mean rainfall for Mahanadi river basin in India. Dabral *et al.* (2014) developed a monthly time series model for pan evaporation for Jorhat (India) adding deterministic and stochastic components. Stochastic dependent component was modelled by MA(2). Dabral *et al.* (2016) developed time series models for rainfall, maximum temperature and minimum temperature for Jorhat (Assam), India by adding deterministic and stochastic components. Stochastic dependent component was modelled using AR(12).

Narayana (1982) reported that a developed time series model is suitable for a certain range and is applicable to the particular zone of climate. The forecasted values based on the time series model have numerous applications namely in the optimal design of soil and water conservation structures, irrigation project planning and studies related to climate change. With this perspective, a study was initiated to develop and validate additive form of time series models for of Doimukh (Itanagar), Arunachal Pradesh (India) for monthly, weekly and daily rainfall of monsoon season.

2. MATERIALS AND METHODS

Study Area and Data Used

Data were collected from Rural Works Department, Arunachal Pradesh (India) which has small meteorological laboratory at Doimukh (longitudes 93º45´05´´E, latitudes 27º08´39´´N and 118 m above mean sea level) where rainfall is recorded on daily basis using Symon's rain gauge. For the present study, recorded daily rainfall data of 29 years (1988-2016) were used. From the available data, the first 20 years (1988-2007) data were used for model development and the remaining data of 9 years (2008-2016) were used for validating the performance of the model.

Time Series Modelling of Deterministic and Dependent Stochastic Component

The additive form of time series model is expressed as:

$$X_t = T_t + P_t + S_t + A_t \quad ...(1)$$

Where, T_t = Deterministic trend component, P_t = Deterministic periodic component, S_t = Stationary stochastic dependent component and A_t = Stationary stochastic independent component t = 1, 2, 3…N, N = total number and X_t = monthly/weekly/daily (monsoon) rainfall data.

The data series of weekly and daily (monsoon season) was transformed by using cube root transformation. This is done for smoothening of the data and stabilising the variance in the selected time series. For detecting the trend in time series, Turning Point test was performed. T_t was estimated through regression method. A trend-free series (Y_t) was obtained as expressed in eq. 2:

$$Y_t = X_t - T_t = P_t + S_t + A_t \quad ...(2)$$

In the series Y_t, periodic component was determined by harmonic analysis. The periodicity of the series Y_t was removed and the standardized stochastic component was obtained using the methodology as discussed in the works of Dabral *et al.* (2008, 2014 and 2016). The stochastic component is constituted by various random effects, which cannot be estimated exactly. In this study, dependent modelling of stochastic component was attempted using autoregressive (AR), moving average (MA), autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) models. Using ISTM2000-V7.1 and SPSS-16 softwares, model parameters, and auto-correlation and partial auto-correlation functions were also computed for the series S_t. Based upon the minimum value of BIC statistics, order of the model was identified.

Modelling of Independent Stochastic Component

The dependent stochastic component was separated from the series. The new series (A_t) was called as an independent stochastic component. For checking of the independence of A_t, the Portmanteau test formulated by McLeod and Li (1983) or Ljung and Box (1978) was used. It was found that the residual series A_t was having some negative values with -2.5, -0.76, and -3.26 as the lowest values of monthly, weekly and daily (monsoon season) rainfall series respectively. Therefore, to convert all values of series A_t, into positive values, an addition of 3, 1, and 4 was made in residual series of monthly, weekly and daily (monsoon season) rainfall respectively. The coefficient of skewness of series, A_t was found to be quite different from zero, therefore, series At is not normally distributed. So, for normalisation, Box-Cox transformation was done. The transformation is mathematically expressed as:

$$y = \frac{x^{\lambda} - 1}{\lambda}; \lambda \neq 0 \quad ...(3)$$

$$y = \ln x \quad ; \lambda = 0.5$$

Where, y is the transformed value of the variate x in the given series a_t, the parameter λ was selected such that the skewness of the transformed data series is minimum. Hinkley (1977) suggested a simpler procedure for the transformation, according to which the test statistic d_λ is calculated for each of the different value of λ varying from -1 to +1 with an interval of 0.1 and coefficient of skewness is computed. The test statistic d_λ is expressed as:

$$d_\lambda = \frac{\bar{y} - \varepsilon_y}{\sigma_y} \quad ...(4)$$

Where, $\bar{y}$ = mean of the transformed series, ε_y = median of the transformed series, and σ_t = standard deviation of transformed series.

The value of λ for which d_λ is minimum was used for the transformation into a normal series.

Independent stochastic component (A_t) was modelled using normal probability distribution function as suggested by Chow *et al.* (1988).The transformed R_t series, in terms of A_t series, is expressed as:

$$R_t = \mu_t + \sigma_t z_t \quad ..... (5)$$

Where, R_t = transformed A_t series, μ_t = mean of the R_t series, σ_t = standard deviation of the R_t series, and z_t = a random component with zero mean and unit variance.

Models Assessment

The models were evaluated using statistical parameters such as absolute error (AE), relative error (RE), correlation coefficient (CC) Nash–Sutcliffe coefficient (E_{NS}), integral square error (ISE) and mean relative error (MRE).

3. RESULTS AND DISCUSSION

Deterministic Modelling

Turning point test was carried out for identification of trend component in the monthly, weekly (cube root transformation) and daily monsoon season rainfall (cube root transformation) series. The value of the P, the E(P) and the Var (P) are given in Table 1. The results indicated the existence of trend component in the monthly, weekly (cube root transformation) and daily monsoon season rainfall (cube root transformation) series as the estimated value of Z obtained through the Turning Point test was found outside the critical range of ±1.96 at 5% level of significance. Thus, no trend hypothesis was rejected.

The trend (T_t) component is given in Table 1. The trend component was removed and a trend free series was obtained for monthly, weekly (cube root transformation) and daily monsoon (cube root transformation) rainfall using eq. 2.

For the development of auto-correlogram, the auto-correlation function r_k of series Y_t for lag 1 to 100 for monthly rainfall, lag 1 to 200 for weekly (cube root transformation) rainfall and lag 1 to 3000 for monsoon (cube root transformation) rainfall was estimated. The auto-correlogram of series Y_t for monthly rainfall, weekly (cube root transformation) rainfall and daily monsoon (cube root transformation) rainfall showed that auto-correlation functions were significantly different from zero, which indicated that all the monthly, weekly (cube root transformation) and daily monsoon (cube root transformation) values of Y_t series were mutually dependent. The peak and trough of the auto-correlogram showed that the series Y_t for monthly rainfall, weekly (cube root transformation) rainfall and daily monsoon rainfall (cube root transformation) had a periodic component with a base period of 12 months, 52 weeks and 153 days, respectively (Fig's 1 to 3).

Fourier decomposition of periodic component and cumulative periodogram of the monthly, weekly (cube root transformation) and daily monsoon (cube root transformation) rainfall series of Y_t at Doimukh are shown in Table 2. It is evident from Table 2 that all 12 harmonics for the monthly rainfall, 10th harmonic for the weekly rainfall and the 8th harmonic for monsoon rainfall explained a total variance of 180.16%, 89.14%, and 79.87%, respectively for Doimukh station. Therefore, other harmonics were ignored. In the

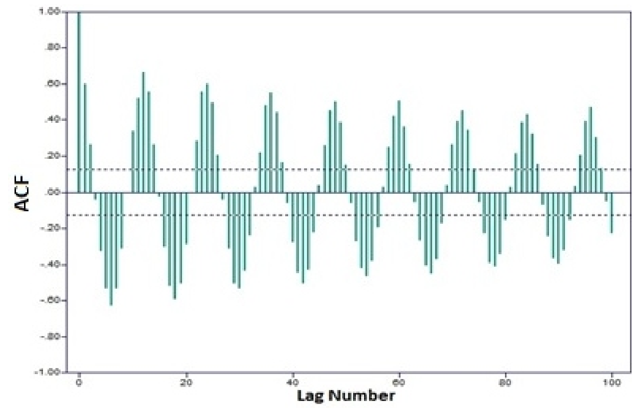

Fig. 1. Auto-correlogram of series (monthly rainfall)

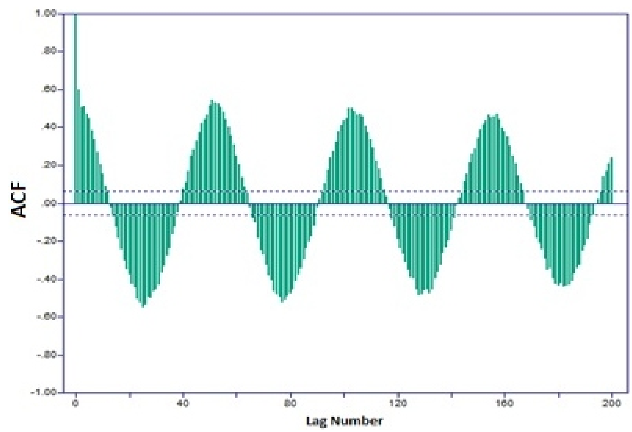

Fig. 2. Auto-correlogram of series of weekly rainfall (cube root transformation)

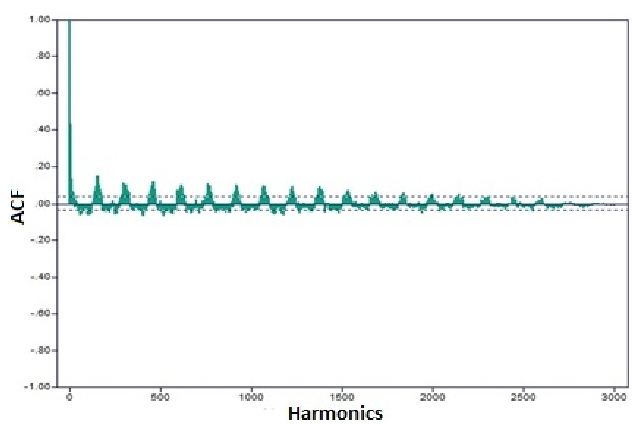

Fig. 3. Auto-correlogram of series of daily monsoon rainfall (cube root transformation)

Table: 1
Analysis of trend component for monthly, weekly and daily monsoon rainfall (Turning point test) and equation of trend line

Variable	Transformation	Turning point test					Equation of trend line
		N	P	E (P)	Var (P)	Z	
Monthly rainfall	Original	240	104	158.6	42.34	-8.40	T_t = -0.3925X + 339.41
Weekly rainfall	Cuberoot	1040	545	692	184.5	-10.8	T_t = -0.0004X + 3.2387
Monsoon	Cuberoot	3060	1179	2038.6	543.6	36.8	T_t = -0.0002X + 1.8639

Table: 2
Fourier decomposition of periodic component of monthly rainfall, weekly rainfall (cube root transformation) and daily rainfall (cuberoot transformation) series (Y_t) at Doimukh

Variable	Order	A_j	B_j	Explained variance (%)	Cumulative explained variance (%)
Monthly rainfall	1	50.990	-39.360	3.100	3.100
	2	45.610	27.870	2.140	5.240
	3	55.200	-7.580	2.320	7.560
	4	-197.740	-294.830	94.280	101.840
	5	154.760	242.000	61.720	163.560
	6	27.090	6.060	0.580	164.140
	7	-7.270	-6.880	0.070	164.220
	8	33.830	-37.330	1.900	166.110
	9	17.240	-65.770	3.460	169.570
	10	-62.610	-19.520	3.220	172.790
	11	31.310	-19.540	1.020	173.810
	12	39.820	-83.120	6.350	180.160
Weekly rainfall (cube root transformation)	1	-0.060	-0.300	1.690	1.690
	2	-0.050	-0.130	0.390	2.090
	3	0.030	-0.050	0.050	2.140
	4	0.040	-0.080	0.170	2.310
	5	0.030	0.050	0.060	2.370
	6	0.010	-0.003	0.000	2.370
	7	-0.010	-0.050	0.040	2.420
	8	0.040	0.040	0.060	2.480
	9	-0.040	0.140	0.400	2.880
	10	-2.130	0.180	86.260	89.140
	11	-0.040	-0.280	1.510	90.640
	12	0.030	-0.094	0.180	90.820
Daily rainfall (cube root transformation)	1	0.029	0.045	0.360	0.360
	2	-0.011	-0.066	0.560	0.920
	3	-0.008	0.050	0.330	1.250
	4	-0.021	-0.004	0.060	1.300
	5	0.079	0.027	0.880	2.190
	6	0.034	-0.011	0.160	2.350
	7	-0.0241	-0.029	0.180	2.520
	8	0.657	-0.427	7.350	79.870
	9	0.0169	-0.015	0.060	79.930
	10	-0.061	0.035	0.620	80.550
	11	0.036	-0.053	0.520	81.070
	12	-0.015	0.013	0.050	81.120
	13	-0.084	-0.062	1.370	82.490

present study, for monthly rainfall, numbers of significant harmonics are different as earlier reported by Dabral *et al.* (2008). Periodic means and periodic standard deviations were also computed. The periodicity of the series Y_t was removed and the standardized stochastic components (S_t) were obtained by the method as explained in the study of Dabral *et al.* (2008 and 2014).The remaining S_t series was subjected to check for stationary behaviour. Once the series was found stationary, it was further subjected for stochastic model identification.

Stochastic Modelling

For monthly rainfall series (S_t), it was observed that the auto-correlation and partial auto-correlation functions at lag 1 to 12 crossed the 95% confidence limit and the values of auto-correlation and partial auto-correlation functions diminished with the increase of lag numbers (Fig's 4 and 5). This indicate that AR, MA, ARMA and ARIMA model may be tried for monthly rainfall series (S_t).

For weekly rainfall (cube root transformation) series (S_t), auto-correlation and partial auto-correlation functions at lag 1 and 2 crossed the 95% confidence limit (Fig,s 6 and 7). Therefore, AR model of order 1 or 2 may be tried for weekly rainfall (cube root transformation) series (S_t).

For daily monsoon rainfall (cube root transformation) series (S_t), auto-correlation and partial auto-correlation functions at lag 1 crossed the 95% confidence limit (Fig's 8 and 9). Therefore, AR model of order 1 may be tried for weekly rainfall (cube root transformation) series (S_t).

As per suggestion of Anderson (1976), MA, ARMA and ARIMA models were also tried for monthly rainfall, weekly rainfall (cube root transformation) and daily monsoon rainfall (cube root transformation) series (S_t) in this study.

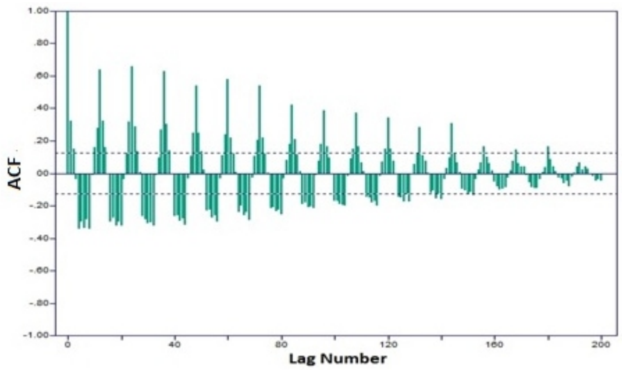

Fig. 4. Auto-correlogram of monthly rainfall series S_t with SE limit

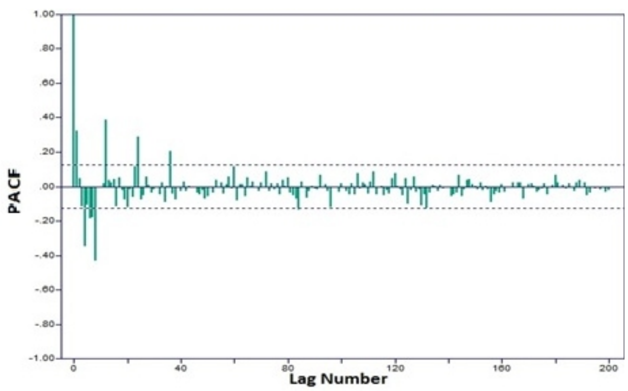

Fig. 5. Partial auto-correlogram of monthly rainfall series S_t with SE limit

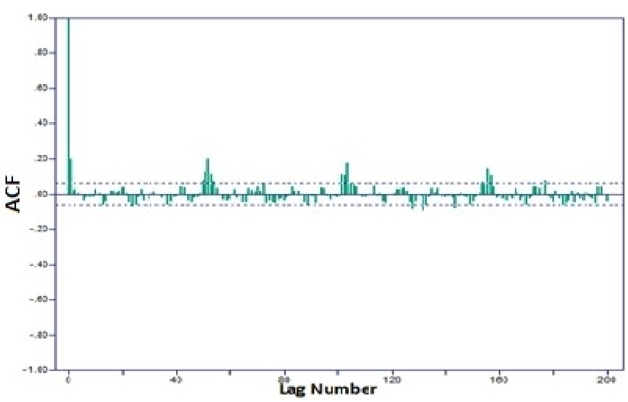

Fig. 6. Auto-correlogram of weekly rainfall (cube root transformation) series S_t with SE limit

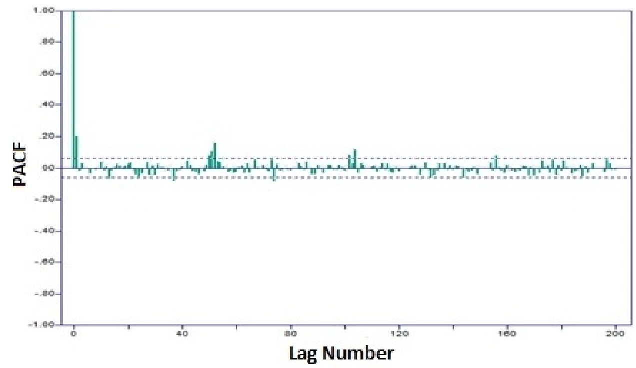

Fig. 7. Partial auto-correlogram of weekly rainfall (cube root transformation) series S_t with SE limit

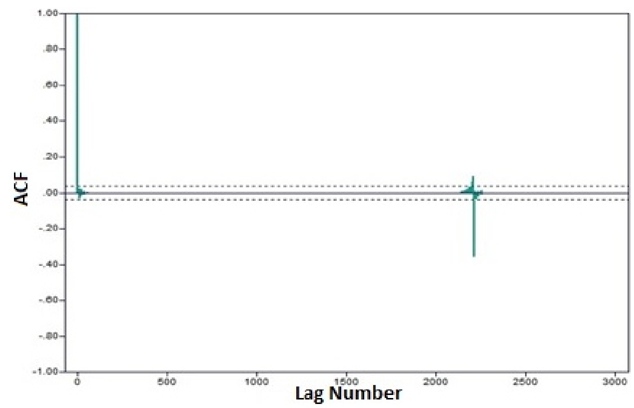

Fig. 8. Auto-correlogram of daily monsoon rainfall (cube root transformation) series S_t with SE limit

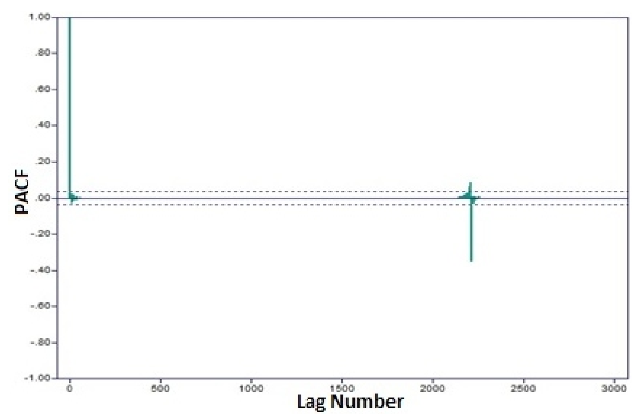

Fig. 9. Partial auto-correlogram of daily monsoon rainfall (cube root transformation) series S_t with SE limit

For monthly rainfall series (S_t), ARMA (4, 8) model best fitted based on the minimum value of BIC statistics. In the present study, for monthly rainfall, the best fit stochastic dependent model is different as earlier reported by Dabral *et al.* (2008). For weekly and daily monsoon rainfall (cube root transformation) series (S_t), AR (1) model best fitted based on the minimum values of BIC statistics (Table 3).

Modelling of Independent Stochastic Component

The calculated values of statistics of Portmanteau test formulated by Ljung and Box (1978) for A_t series of monthly rainfall, weekly (cube root transformation) rainfall and daily monsoon (cube root transformation) rainfall series were found to be 30.45, 31.38 and 13.461, respectively, which are less than the tabular value of chi-square value (35.020) at 5% significant level at 20 degree of freedom. This indicated that A_t series of the independent stochastic component for monthly, weekly (cube root transformation) and daily monsoon (cube root transformation) rainfall consist of independent and identically distributed variables. Before modelling the independent stochastic component, the normality of series A_t was checked by finding the values of

Table: 3
The best fit autoregressive models for series S_t

Variable	Transformation	Best fit model	Description of model	BIC
Monthly rainfall	Original	ARIMA (4,0,8)	$S_t = 0.994\ S_{t-2} - 0.997\ S_{t-4} - 1.11\ R_{t-2} + 0.955\ R_{t-4} - 0.186\ R_{t-8}$	-0.551
Weekly rainfall	Cube root	AR (1)	$S_t = 0.205\ S_{t-1}$	-0.036
Monsoon	Cube root	AR (1)	$S_t = 0.007\ S_{t-1}$	0.003

coefficient of skewness, since a normal distribution has a coefficient of skewness equal to zero. It was found that the residual series A_t was having some negative values with -2.5, -0.76, and -3.26 as the lowest values of monthly, weekly and daily (monsoon season) rainfall series, respectively therefore, to convert all values of series, at, into positive values, an addition of 3, 1, and 4 was made in residual series of monthly, weekly and daily (monsoon season) rainfall, respectively. The coefficient of skewness was found to be -2.35, -0.8 and -2.75 in A_t residual series of monthly, weekly and daily monsoon season rainfall, respectively which is quite different from zero, therefore, series A_t were not normally distributed series. To obtain a normalized form of series A_t, Box-Cox transformation was applied as discussed in section 2.6. According to that test, d_λ, is calculated for each of different value of λ varying from -1 to +1 with an interval of 0.1. Coefficient of skewness was computed and the results are given in Table 4.

As discussed above, normal probability distribution function was fitted for modelling the independent stochastic component to monthly rainfall, weekly (cube root transformation) rainfall and daily monsoon (cube root transformation) rainfall series (A_t) obtained after Box-Cox transformation. The estimated χ^2 value was found to be less than that of table value at 5% level of significance in all cases. After substituting transformed residual series A_t in eq. 5, the series was expressed in the form of eq. 9. The details of results are presented in Table 4.

Time Series Models

Time series models for monthly, weekly (cube root transformation) and daily monsoon (cube root transformation) rainfall series were developed by adding the values of deterministic and stochastic components in eq. 1. The decomposition models of the time series (X_t) are given in Table 5.

Model Assessment

Monthly rainfall series

The mean monthly values of rainfall from 1988 to 2007 of observed and predicted data are presented in Table 6. The mean and standard deviation of the predicted series (1988-2007) are found to be 348.7 mm and 415.5 mm, which is close to the mean of 312.8 mm and standard deviation of 393.0 mm of the observed series. The value of absolute error (AE) was found in the range of 38.3 mm to 401.4 mm. It was found low in the months of January, April, November and December. The value of relative error (RE) was found to be low in the months April, May, July, August and September (Table 6). The values of correlation coefficient (CC) and Nash-Sutcliff (E_{NS}) were 0.83 and 0.69, respectively (Table 6). In order to further evaluate the model performance, ISE and MRE values were also computed and their values were found to be 0.051 and 0.0002, respectively which indicated the developed model fitted well (Table 6).

Weekly rainfall (cube root transformation) series

The mean and standard deviation of the predicted series (1988–2007) are found to be 3.3 mm and 1.5 mm, which is close to the mean of 3.0 mm and standard deviation of 1.5 mm of the observed series. The mean weekly rainfall (cube root transformation) values from 1988 to 2007 of observed and predicted data are shown in Fig. 10. The values of respective RE and AE errors of all the weeks were estimated and presented in Fig.10. The value of AE error was found in the range of 0.1 mm to 1.2 mm. The values of RE were found to be comparatively higher for dry weeks *i.e.* week no. 1 to 7, 9 to 11, and 43 to 52.The values of CC and E_{NS} were observed to be 0.79 and 0.596, respectively (Fig. 10). In order to further evaluate the model performance, the values of ISE and MRE were also computed, and their values were found to be 0.013 and 0.0001, respectively which indicated the developed model fitted well (Fig.10).

Table: 4
Box-Cox transformation statistics χ^2 value and transformed residual series

Variable	Addition in residual series	Optimal value of λ	d_λ	μ_t	σ_t	Skewness coefficient	χ^2 value	Transformed residual series
Monthly rainfall	3	-1	-0.27	2.57	0.94	-2.35	12.91	$a_t = \{1/[-1(\mu_t+\sigma_t z_t)+1]\}-3$
Weekly rainfall (cube root transformation)	1	-1	-0.20	0.1	0.18	-0.8	10.3	$a_t = \{1/[-1(\mu_t+\sigma_t z_t)+1]\}-1$
Monsoon rainfall (cube root transformation)	4	-1	-0.17	0.716	.119	-2.75	18.45	$a_t = \{1/[-1(\mu_t+\sigma_t z_t)+1]\}-4$

Table: 5
Decomposition of time series models

Variable	Decomposition of model
Monthly rainfall	$X_t = (-0.393X+339.410)+(0.003)+[(50.990)\cos(\frac{2\pi}{12})\tau+(-39.360)\,in(\frac{2\pi}{12})\tau+(45.600)\cos(\frac{4\pi}{12})\tau+(27.866)\sin(\frac{4\pi}{12})\tau+(55.200)\cos(\frac{6\pi}{12})\tau+(-7.580)\sin(\frac{6\pi}{12})\tau+(-197.742)\cos(\frac{8\pi}{12})\tau+(294.831)\sin(\frac{8\pi}{12})\tau+(154.7612)\cos(\frac{10\pi}{12})\tau+(241.996)\sin(\frac{10\pi}{12})\tau+(27.090)\cos(\frac{12\pi}{12})\tau+(6.056)\sin(\frac{12\pi}{12})\tau+(7.2687)\cos(\frac{14\pi}{12})\tau+(6.879)\sin(\frac{14\pi}{12})\tau+(33.826)\cos(\frac{16\pi}{12})\tau+(37.333)\sin(\frac{16\pi}{12})\tau+(17.238)\cos(\frac{18\pi}{12})\tau+(65.769)\sin(\frac{18\pi}{12})\tau+(62.612)\cos(\frac{20\pi}{12})\tau+(19.522)\sin(\frac{20\pi}{12})\tau+(31.306)\cos(\frac{22\pi}{12})\tau+(-19.538)\sin(\frac{22\pi}{12})\tau+(39.819)\cos(\frac{24\pi}{12})\tau+(-83.118)\sin(\frac{24\pi}{12})T]+(0.994\,S_{t-2}-0.997\,S_{t-4}-1.11\,R_{t-2}+0.955\,R_{t-4}-0.186\,R_{t-8})+((-1(2.570+0.94\,Z_t)+1)^{-1}-3)$
Weekly rainfall (cube root transformation)	$X_t=(-0.0004\,X+3.239)+(-0.016)+[(-2.133)\cos(\frac{20\pi}{12})\tau+(0.181)\sin(\frac{20\pi}{12})\tau]+(0.205\,S_{t-1})+((-1(0.1+0.18\,Z_t)+1)^{-1}-1)$
Daily monsoon rainfall (cube root transformation)	$X_t=(-0.0002\,X+1.864)+(-0.016)+(0.657)\cos(\frac{16\pi}{12})\tau+(-0.427)\sin(\frac{16\pi}{12})\tau+(0.007\,S_{t-1})+((-1(0.716+0.119\,Z_t)+1)^{-1}-4)$

Table: 6
Mean monthly rainfall of observed and predicted data series (1988-2007) along with errors

Month	Mean of observed data (mm)	Mean of predicted data (mm)	Errors		Integral square error	Mean relative error	Correlation coefficient	Nash-Sutcliff coefficient
			Absolute error (mm)	Relative error (%)				
January	38.3	0.0	38.3	100.0	0.051	0.0002	0.830	0.690
February	79.7	0.0	79.7	100.0				
March	112.5	27.8	84.7	75.3				
April	236.0	195.0	41.0	17.4				
May	497.6	657.3	159.7	32.1				
June	768.8	1170.2	401.4	52.2				
July	622.5	875.7	253.2	40.7				
August	477.6	641.6	163.9	34.3				
September	435.4	581.5	146.0	33.5				
October	194.4	35.3	159.1	81.8				
November	31.0	0.0	31.0	100.0				
December	11.5	0.0	11.5	99.9				

Daily monsoon rainfall (cube root transformation) series

The mean and standard deviation of the predicted series (1988-2007) were found to be 1.63 mm and 0.33 mm, which are close to the mean of 1.59 mm and standard deviation of 0.63 mm of the observed series. The mean monthly values of rainfall from 1988 to 2007 of observed and predicted data are shown in Fig. 11. The value of AE error was found in the range of 0.01 mm to 0.90 mm. The value of RE (%) was found to in the range of -0.03 to 349.60 (Fig.11). The values of CC and E_{NS} were observed to be 0.74 and 0.50, respectively (Fig.11). In order to further evaluate the performance, the values of ISE and MRE were also computed and their values were found to be 0.04 and 0.0000004, respectively which indicated the developed model fitted well (Fig.11).

Validation of the Developed Time Series Models

Monthly rainfall series

Time series model was used for prediction of 9 years value of monthly rainfall *i.e.* for the years 2008 and 2016. The predicted value of monthly rainfall alongwith the observed values for the year 2008 and 2016 are shown in Fig. 12. The mean of the predicted monthly rainfall series was 307.3 mm whereas the mean of observed series was 174.2 mm. The mean monthly values of rainfall (2008 to 2016) of the predicted and observed values are shown in Table 7. The value of AE was found to be in the range of 8.6 mm to 656.4 mm. The value of RE was found to be low in the month of April and August only. The values of CC and E_{NS} were observed to be 0.766 and 0.59 (Table 7). In order to further evaluate the performance, the values of ISE and

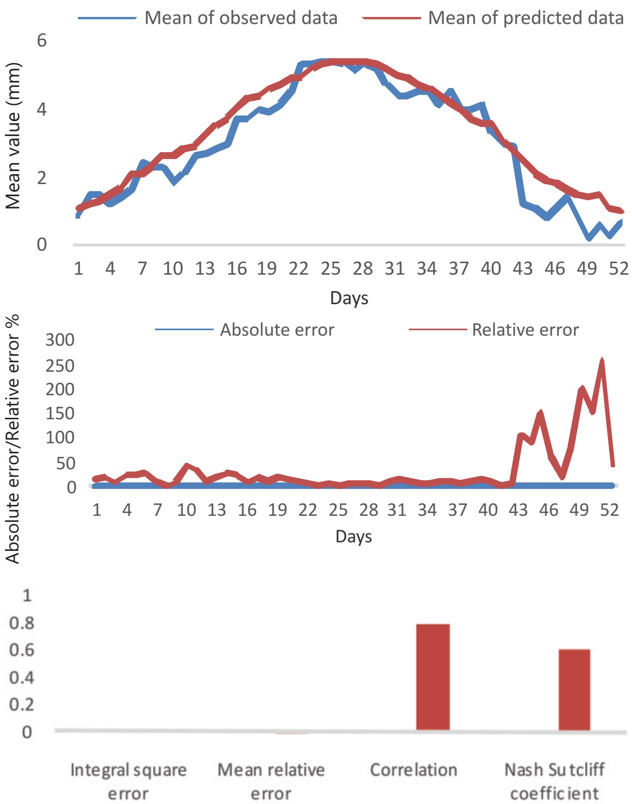

Fig. 10. Mean weekly rainfall (cube root transformation) of observed and predicted data (1988-2007) alongwith errors

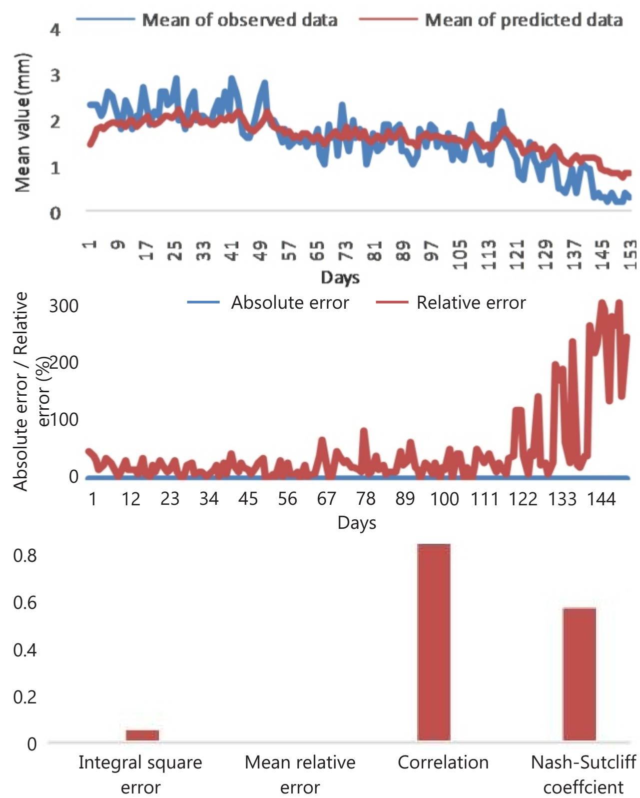

Fig. 11. Mean daily monsoon rainfall (cube root transformation) of observed and predicted data (1988-2007) alongwith error

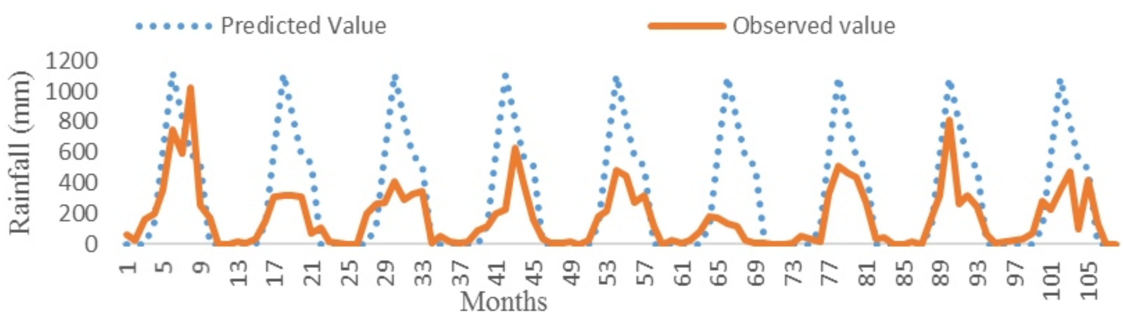

Fig. 12. Observed and predicted value of monthly rainfall for the years (2008-2016)

Table: 7
Mean monthly rainfall of observed and predicted data series (2008-2016) alongwith errors

Month	Mean of observed data (mm)	Mean of predicted data (mm)	Errors		Integral square error	Mean relative error	Correlation	Nash-Sutcliff coefficient
			Absolute error (mm)	Relative error (%)				
January	16.1	0.0	16.1	100.0	0.108	0.001	0.770	0.590
February	20.6	0.0	20.6	100.0				
March	78.2	0.0	78.2	100.0				
April	175.7	126.3	49.4	28.1				
May	266.7	588.4	321.7	120.6				
June	444.9	1101.3	656.4	147.5				
July	400.2	807.3	407.1	101.7				
August	356.8	573.8	216.9	60.8				
September	232.8	514.0	281.2	120.8				
October	75.3	0.0	75.3	100.0				
November	14.4	0.0	14.4	100.0				
December	8.6	0.0	8.6	100.0				

MRE were also computed and their values were found to be 0.0007 and 0.108, respectively which indicated developed model fitted well (Table 7).

Weekly rainfall (cube root transformation) series

Time series model was used for prediction of 6 years value of weekly rainfall (cube root transformation) *i.e.* for the years 2008 and 2013. The predicted values of weekly rainfall (cube root transformation) alongwith the observed values for the year 2008 and 2016 are shown in Fig. 13. The mean of the predicted weekly rainfall was 2.9 mm which was close to the mean value of 2.40 mm of the observed value. The mean weekly values of rainfall (cube root transformation) (2008 to 2016) of the predicted and observed values are shown in Fig. 14. The value of AE was found in the range of 0.03 mm to 1.56 mm. The value of RE was found in the range of 0 to 433.4 %.The values of CC and E_{NS} were observed to be 0.77 and 0.50 (Fig.14). In order to further evaluate the performance, the values of ISE and MRE were also computed and their values were found to be 5.521×10^{-5} and 0.0097 which indicated developed model fitted well (Fig.14).

Daily monsoon rainfall (cube root transformation) series

Time series model was used for prediction of 6 years *i.e.* for the years 2008 and 2016. The predicted values of daily monsoon rainfall along with the observed values for the year 2008 and 2016 are shown in Fig. 15. The annual mean of the predicted monsoon rainfall series was 1.89 mm which was close to the mean value of 1.88. The mean daily monsoon values of rainfall (cube root transformation) (2008 to 2016) of the predicted and observed values are shown in Fig. 16. The absolute error was found in the range of 0 mm to 15 mm. Relative error was found in the range of 0 to 372.9% (Fig.16).The values of CC and E_{NS} were observed to be 0.80

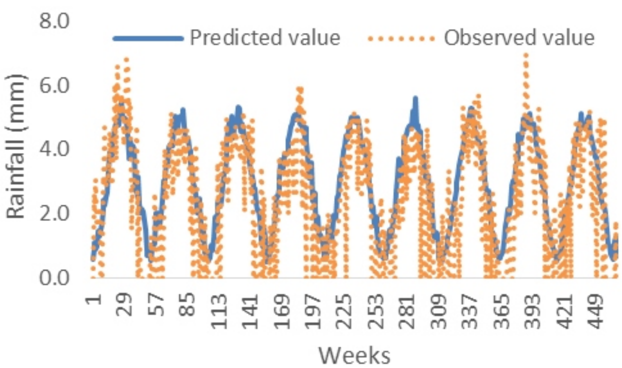

Fig. 13. Observed and predicted value of weekly rainfall for the years (2008-2016)

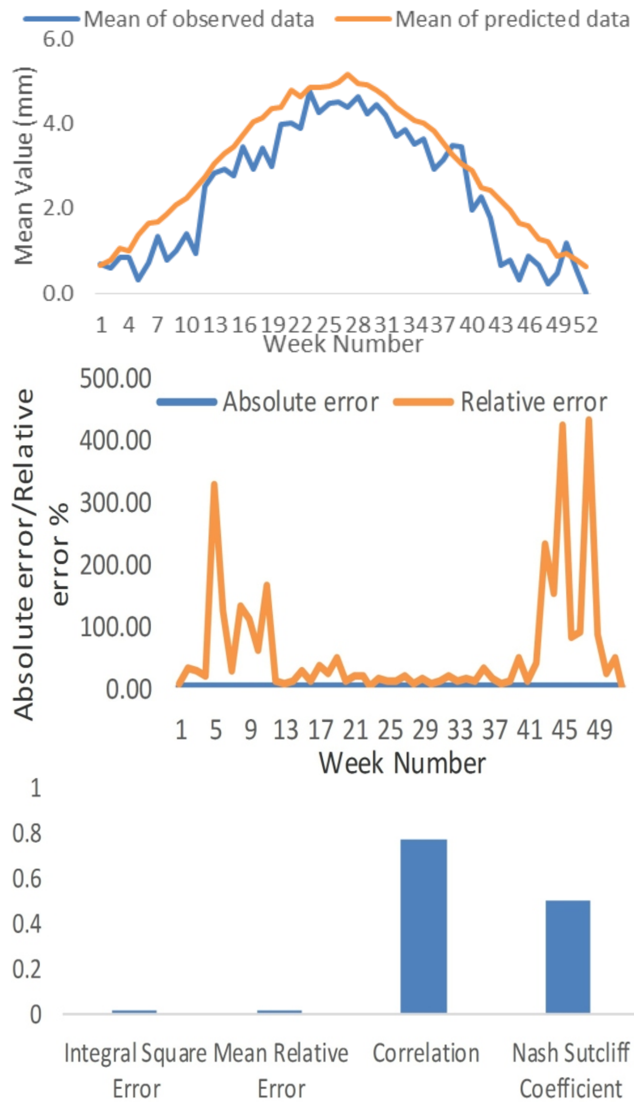

Fig. 14. Mean weekly rainfall of observed and predicted data (2008-2016) alongwith errors

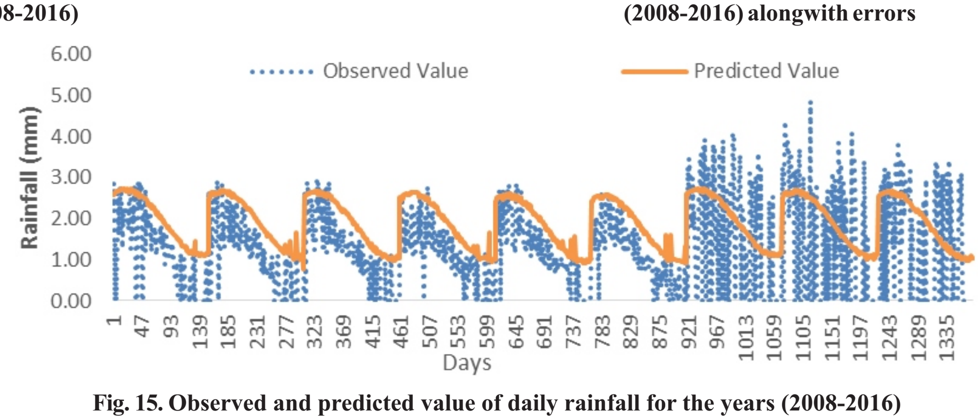

Fig. 15. Observed and predicted value of daily rainfall for the years (2008-2016)

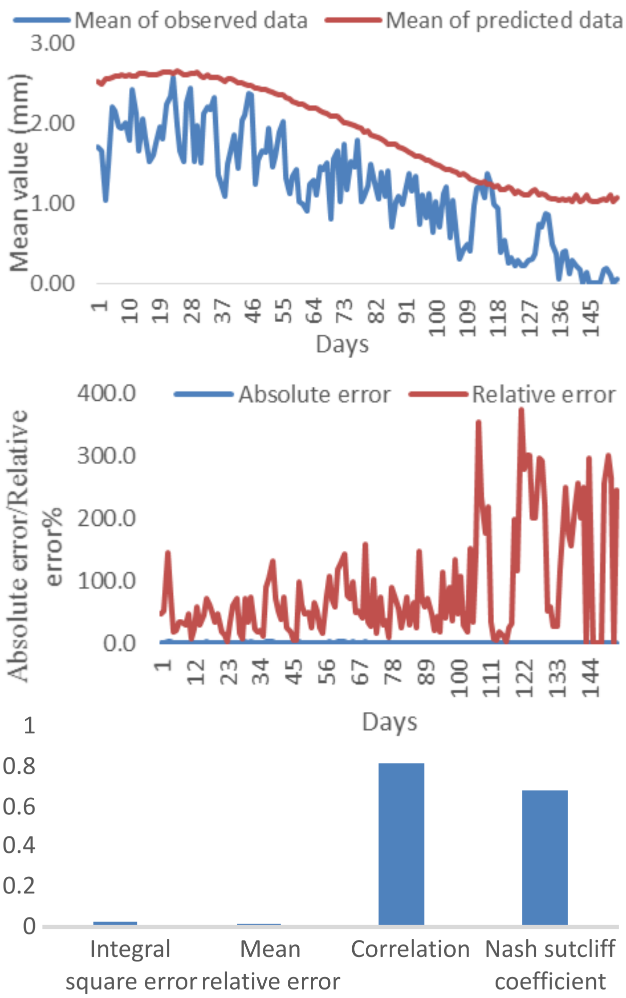

Fig. 16. Mean daily monsoon rainfall of observed and predicted data (2008-2016) alongwith errors

and 0.66 (Fig. 16). In order to further evaluate the performance, the values of ISE and RE were also computed and their values were found to be 0.025 and 0.00035 respectively which indicated developed model fitted well (Fig.16).

4. CONCLUSIONS

Monthly rainfall, weekly rainfall (cube root transformation) and daily monsoon rainfall (cube root transformation) series were having trend, periodic and stochastic components. The mean and standard deviation of the predicted value was found in close agreement to the observed value. Other selected estimated statistical parameters also indicated good fitness of models. Developed models can be used for short term forecast or synthetic data generation for the study area.

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