



# Flume experiment for evaluation of effect of different bed conditions on Manning's roughness coefficient in open channel flow

### Pragati N. Sawant<sup>1,\*</sup>, Akhilesh Kumar<sup>2</sup> and Shreya Nivesh<sup>2</sup>

<sup>1</sup>Department of Soil and Water Conservation Engineering, Dr Annasaheb Shinde College of Agricultural Engineering and Technology, Mahatma Phule Krishi Vidyapeeth, Rahuri, Ahmednagar - 413722, Maharashtra; <sup>2</sup>Department of Soil and Water conservation Engineering, College of Technology, Govind Ballabh Pant University of Agriculture Technology, Pantnagar - 263145, Uttarakhand.

\*Corresponding author:

E-mail: pragatisawant2012@gmail.com (Pragati N. Sawant)

#### **ARTICLE INFO**

Article history:					
Received	:	November, 2018			
Davisad		July 2020			

Revised : July, 2020 Accepted : August, 2020

#### **Key words:** Gravel bed Manning's roughness coefficient Open channel flow

#### 1. INTRODUCTION

An understanding of flow resistance is the most basic knowledge required by engineers. For practical applications, flow resistance is typically quantified by roughness coefficient. Since the roughness coefficient has an extensive effect on flow analysis of a river, including computation of the water level and velocity, its accurate estimation is important for prediction of the water level during flooding, design of hydraulic structures, and stability assessment of revetments. In addition, considering ecological aspects of a river, the roughness coefficient is also significant in simulation of flow conditions associated with habitat suitability. Relationships such as the Manning equation, Chézy formula and Darcy-Weisbach equation have been in use for a century or more. All of them account for resistance with a single coefficient of resistance. The central problem is evaluation of this coefficient. Efforts have been made by various researchers to quantify the roughness coefficients of gravelbed rivers in an objective manner because of its importance. Among them, an element-based method (Cowan, 1956) and

#### ABSTRACT

The direct measurement of stream bed surface roughness in terms of roughness coefficients remains a difficult task. The variation in Manning's roughness coefficient (n) occurs due to many contributing factors, which shows necessity to determine n for different bed materials. This study was carried out with objective to determine Manning's roughness coefficient for different sizes of bed materials using selected discharges and bed slopes. Experiments were conducted in a hydraulic flume by creating different bed conditions under controlled conditions in the laboratory. Discharges were varied from 0.088 m<sup>3</sup>s<sup>-1</sup>m<sup>-1</sup> to 0.038 m<sup>3</sup>s<sup>-1</sup>m<sup>-1</sup> with varying bed slopes from 0.25% to 2%. The value of Manning's roughness coefficient was found to be increasing with the increasing size of bed material and bed slope for every discharge. A mathematical relationship was developed in terms of bed slope, discharge and gravel size with help of the experimental findings. Different types of performance indicators *viz.*, coefficient of determination (r<sup>2</sup>) and coefficient of efficiency (CE) were used to evaluate the accuracy of the developed model. The results indicate better agreement between observed data and predicted results for the newly developed parameterized equation.

empirical equations that relate the roughness coefficient either to bed material (Strickler, 1923; Meyer-Peter and Muller, 1948; Bray, 1979) or to relative depth of flow (Bray, 1979; Limerinos, 1970) are found to be representative. Barnes (1967) stated that hydraulic computations involving flow in open channels require an evaluation of the roughness characteristics of the channel and stated that in the absence of a satisfactory quantitative procedure this evaluation remains chiefly an art.

In open channels, Manning formula has been widely used to determine the roughness coefficient, n. There is a tendency to regard the selection of roughness coefficients as either an arbitrary or an intuitive process. Specific procedures can be used to determine the values for roughness coefficient in channels and flood plains. The n values for channels are determined by evaluating the effects of certain roughness factors in the channels. The variation of roughness coefficient occurs due to many contributing factors, such as, surface roughness, nature of surface vegetation, flow depth, channel irregularity, channel alignment, silting and scouring, obstruction and bed slope. A proper vegetative cover would also reduce runoff velocity and enhance opportunity time for infiltration Mishra et al. (2018). Obstructions, such as logs, stumps, boulders, debris, pilings, and bridge piers disturb the flow pattern in the channel and increase roughness. The amount of increase depends on the shape of the obstruction; the size of the obstruction in relation to that of the cross section; and the number, arrangement, and spacing of obstructions. Several obstructions can create overlapping spheres of influence and may cause considerable disturbance, even though the obstructions may occupy only a small part of a channel cross section. Chow (1959) assigned adjustment values to four levels of obstruction: negligible, minor, appreciable, and severe. Some other researchers have focused on differentially roughened channels, using artificial uniform roughening materials on the boundary, e.g. Tzelepis et al. (2015) or using natural gravels on the bed e.g. Zeng et al. (2015). Channel roughness seems to be directly related to channel gradient or slope (Jarrett, 1985). Channels with low gradients have been shown to have lower roughness coefficients than channels with high gradients (Jarrett, 1985). Lopez and Barragan (2008) investigated an equivalent roughness of gravel-bed rivers. The relation between the equivalent roughness and different grain size percentiles of the sediment in gravel-bed rivers was determined. However, there are few studies focusing on the flow resistance in composite or uniformly roughened rectangular channels with non-mobile boundaries.

As per above, in the present study, efforts have been made to quantify the effects of different sizes of bed materials under varying discharges and bed slope conditions on Manning's surface roughness coefficient. The objective of the present work is to investigate the direct dependence of Manning's roughness on the different sizes of gravel materials. Thus, using the available data obtained through laboratory campaigns, an equation to assess the Mannings roughness coefficient knowing the slope, discharge and gravel size is proposed in the study.

#### 2. MATERIALS AND METHODS

#### **Experimental Setup**

The experimental setup used in this study was available in the Soil and Water Conservation Engineering laboratory of the Department of Soil and Water Conservation Engineering. The experimental setup basically consisted of a rectangular flume with water circulation and measurement system. Various components of the experimental setup are described below.

#### **Tilting hydraulic flume**

The tilting hydraulic flume used in this study was 7 m  $\times$  0.60 m  $\times$  0.3 m size, which had 1 m long and 0.60 m deep settling chamber at its upstream end. The testing section of the flume was 6 m long. Both sides of flume were provided

with thick transparent perspex/glass sheet of 3.6 m long to facilitate visual observation of flow pattern along the flume. The bed and part of walls at both ends were made up of steel material. The inlet section of the flume was provided with 2 numbers of baffles for stream lining the flow. The flume had two adjustable gates with rack and pinion arrangement and was provided with a pipe railing in flume length between the gates for movement of pointer gauge trolley. In the present case, flume was provided with a maximum forward slope of 5%. It could also be adjusted at an adverse slope for a maximum up to 1%. A 5 hp mono block three phase electric operated centrifugal pump was used to supply water to the flume unit, through a pipe line. The pump received water from storage tank downstream of the experimental setup. The experimental setup is shown in Fig. 1.

#### **Experimental Techniques**

#### **Preparation of bed material**

As per requirement of the experiment, different sizes of the bed material were needed which was available in the concrete laboratory of Department of Civil Engineering. Three different sizes of bed material were selected for use and they needed to be prepared by grading or sorting using the sieves available. Four sieves of following sizes were taken-4.75 mm, 10 mm, 20 mm, and 30 mm. To get material of 10 mm size, it was passed through the 10 mm size sieve which was retained on the sieve of 4.75 mm size. Similarly, for 20 mm size, bed material was passed through 20 mm size sieve and the material was retained on the 10 mm size sieve. To sort material of 30 mm size, bed material was passed through the sieve of 30 mm size and it was retained on the 20



Fig. 1. Experimental setup

mm size sieve. The gradation work was done manually in the concrete laboratory.

#### **Discharge measurement and computation**

In order to obtain the discharge, manometer was connected to the two outlet points which were provided across the orifice plate in the supply line. Before starting the actual computation of discharge, the manometer was made bubble free which was done manually, a provision which is provided in the manometer itself. Before the start of the experiment, the level of mercury in the manometer is equal on both sides. As flow passed through the pipe, the difference in the level of the two sides of manometer can be seen. As the pressure increases, the differential head increases and with it the discharge also increases and vice-versa. In order to get the same discharge for different experimental runs, the differential head was kept to be the same, so that the same can be obtained for further experiments. The differential head was recorded, and by using the formulae given below discharge was computed.

$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \sqrt{2gh} \qquad \dots (1)$$

The discharge is computed using eq.1 of orifice meter where, Q is discharge in lit/sec,  $a_0$  and  $a_1$  are area of orifice and area of pipe, respectively.

$$h = x \left( \frac{S_g}{S_0} - 1 \right) \tag{2}$$

Where, *h* is the differential head in cm, *x* is the manometer readings in cm,  $S_g$  is specific gravity of mercury and  $S_o$  is specific gravity of water as shown in eq. 2.

#### Estimation of Manning's roughness coefficient

After establishment of a uniform flow, readings for flow depth were recorded by using point gauge. Point gauge was moved along the flume length and the readings were taken at the mid-point of the flume. For smooth bed condition, readings for flow depths were obtained for three different discharges  $0.088 \text{ m}^3 \text{s}^{-1} \text{m}^{-1}$ ,  $0.053 \text{ m}^3 \text{s}^{-1} \text{m}^{-1}$  and  $0.038 \text{ m}^3 \text{s}^{-1} \text{m}^{-1}$  and seven different slopes 0.25%, 0.33%, 0.66%, 1%, 1.33%, and 2% for each discharge. For each bed condition, the slope was changed using the mechanism provided in the tilting flume.

After placing the bed material, water was released into the flume and allowed to get a uniform flow so that the flow depth readings for the same combinations of discharges and bed slopes could be measured. Similarly, identical procedures were followed for gravel sizes of 10 mm, 20 mm, and 30 mm. Thus, a total of 84 experimental runs were performed for the Manning's roughness coefficient determination. After the recording of all the readings by using the formulae stated below, the value of Manning's roughness coefficient was determined. The amount of water passing a point on the channel during a given time is a function of velocity and crosssectional area of the flowing water. Flow velocity was calculated using formula for continuity equation as follows:

$$Q = A \times V \qquad \dots (3)$$

Where, Q is discharge rate (volume/time), A is crosssectional area, and V is flow velocity as shown in eq. 3. A resistance formula proposed by Irish Engineer, for uniform flow in open channels, is:

$$V = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \dots (4)$$

Where, V is flow velocity in m s<sup>-1</sup>, n is Manning's roughness coefficient, A is channel cross-sectional area in square meter, R is hydraulic radius (cross-sectional area / wetted perimeter) in meter, and S is slope in % as shown in eq. 4. Wetted perimeter is the length across the channel where the stream water is in contact with the bed and the side walls.

#### Analysis of observed data

Analysis of observed data was performed to calculate the value of Manning's roughness coefficient for different combinations of bed material, bed slope and discharge. Once the observed values of Manning's roughness coefficient for different combinations were obtained, a comparative analysis was performed to see the effect of bed material, bed slope and discharge on roughness coefficient. Also attempts was made to develop a relationship between Manning's roughness coefficient, bed material, bed slope and discharge. Using the developed relationship, the values for 'n' were computed and compared with the observed values. In this study, simple multiple linear regressions (MLRs) were used to obtain the predicted values and the mathematical relationships in terms of involved variables.

#### Multiple linear regression (MLR)

The term MLR was first used by Pearson in 1908. MLR takes a group of random variables or samples and tries to find a mathematical relationship between the taken variables. The model develops a relationship in the form of a straight line (linear) that best approximates all the individual data points of the study conducted. The general purpose of multiple regression analysis was to learn more about the relationship between several independent or predictor variables in the study and a dependent or criterion variable of the study. Once regression line has been obtained, graph of the expected (predicted) and the observed values can be easily constructed.

Regression analysis is incorporated in the study when two or more variables are thought to be systematically connected by a linear relationship. MLR applies to problems in which records have been kept of one variable, y, the dependent variable, and several other variables  $x_1, x_2, ..., x_k$ , the independent variables, and in which the objective requires the relationship between the variable y and the variables  $x_1, x_2, ..., x_k$  to be investigated. In the present study, the MLR analysis was performed on the same data set to estimate Manning's roughness coefficient and the regression equation used is defined as:

$$n = \alpha_1 + \alpha_2 S + \alpha_3 q + \alpha_4 D \qquad \dots (5)$$

Where,  $\alpha_{1}$ ,  $\alpha_{2}$ ,  $\alpha_{3}$ , and  $\alpha_{4}$  are constants and *S*, *D*, and *Q* are the variables as shown in eq. 5. Thus, it is assumed that n is linearly related to each of the independent variables.

#### Performance assessment of developed model

Qualitative and quantitative evaluation of developed model is performed to judge the goodness of fit between measured and predicted values. The parameters used were namely integral square error (ISE), coefficient of efficiency (CE) and index of agreement.

#### Integral square error (ISE)

The ISE, another measure of goodness of fit between the observed and predicted values is in fact proportion to the ratio of the root mean square error to sum of observed Manning's roughness coefficient. The ISE is calculated by the following relationship as proposed by Diskin *et al.* (1978).

$$ISE = \frac{\sqrt{\left[\sum_{i=1}^{k} \left(n_{i} - \hat{n}_{i}\right)^{2}\right]}}{\sum_{i=1}^{k} n_{i}} \times 100 \qquad \dots (6)$$

Where,  $n_i$  is the  $i^{\hbar}$  observed value of Manning's roughness coefficient,  $\hat{n}_i$  is the corresponding predicted value of Manning's roughness coefficient, k is total number of observations as shown in eq. 6.

#### Coefficient of efficiency (CE)

The CE developed by Nash and Sutcliffe (1970) is an improvement over  $R^2$  statistic. It is analogous to coefficient of determination  $(R^2)$  in linear regression but not identical. The coefficient of determination indicates the level of variance explained by the model. It gives the proportions of variance of the observation accounted for by the model.

Nash-Sutcliffe coefficient of efficiencies can range on a scale of - $\infty$  to 1. An efficiency of 1 (CE = 1) corresponds to a perfect match between model predictions and observations. Similarly, an efficiency of 0 indicates that the model predictions are as accurate as the mean of the observed data series, whereas an efficiency less than zero (- $\infty$  < CE < 0) occurs when the observed mean is a better predictor. It is expressed mathematically as shown in eq. 7:

$$CE = 1 - \frac{\sum_{i=1}^{k} {\binom{n_{i} - n_{i}}{\binom{k}{2}}}^{2}}{\sum_{i=1}^{k} {\binom{n_{i} - n_{i}}{2}}^{2}} \qquad \dots (7)$$

A value of Nash-Sutcliffe coefficient of efficiency in the range of 0.9 and above is generally considered very satisfactory, 0.8–0.9 represents a fairly good prediction model, and below 0.8 is considered unsatisfactory (Shamseldin, 1997).

#### Index of agreement (d)

The Index of Agreement (d) was developed by Willmott (1981) as a dimensionless standardized measure of the degree of model prediction error and varies between 0 and 1. A value of 1 indicates a perfect match, and 0 indicates no agreement at all. It was not designed to be a measure of correlation but of the degree to which a model's predictions are error free. The index of agreement can detect additive and proportional differences in the observed and simulated means and variances; however, it is overly sensitive to extreme values due to the squared differences (Legates and McCabe, 1999). Index of agreement (d) is determined using the following relationship as shown in eq. 8.

$$d = 1 - \frac{\sum_{i=1}^{k} \left( n_{i} - n_{i} \right)^{2}}{\sum_{i=1}^{k} \left( \left| \hat{n}_{i} - n_{i} \right| + \left| n_{i} - n_{i} \right| \right)^{2}} \qquad \dots (8)$$

#### 3. RESULTS AND DISCUSSION

#### Manning's Roughness Coefficient for Different Bed Conditions using Selected Slopes at a Particular Discharge

Presentation of the observations done graphically in Fig. 2a, b, c, d for smooth bed, 10 mm gravel bed, 20 mm gravel bed, and 30 mm gravel bed conditions indicates that for all selected discharges, as bed slope increases there is increase in value of Manning's roughness coefficient. However, the rate of increase is not linear as value of bed slope is increasing for corresponding discharges. When the bed slope increased from 0.25% to 2.00% the value of Manning's roughness coefficient is increasing exponentially as shown in Fig. 2a, b, c, d.

It was also observed from recorded data (Table 1) that for the bed slope of 0.25% while discharge increased from  $0.038 \text{ m}^3 \text{s}^{-1} \text{m}^{-1}$  to 0.088  $\text{m}^3 \text{s}^{-1} \text{m}^{-1}$ , the value of Manning's roughness coefficient decreased from 0.0097 to 0.00865 for smooth bed condition. Similar trend of decreasing Manning's roughness coefficient was found for the all other rough bed conditions.

Similarly, for the bed slope of 1.00%, when discharges were kept as  $0.088 \text{ m}^3 \text{s}^{-1} \text{m}^{-1}$ ,  $0.053 \text{ m}^3 \text{s}^{-1} \text{m}^{-1}$  and  $0.038 \text{ m}^3 \text{s}^{-1} \text{m}^{-1}$ , the Manning's roughness coefficient was found to be 0.0093, 0.0101 and 0.0110, respectively. It was observed that when discharge decreased from 0.088  $\text{m}^3 \text{s}^{-1} \text{m}^{-1}$  to 0.038  $\text{m}^3 \text{s}^{-1} \text{m}^{-1}$ , the Manning's roughness coefficient 'n' was observed to increase from 0.0095 to 0.0123 for the bed slope of 2.00%. It can be concluded that for a discharge when bed slope



Fig. 2. Manning's roughness coefficient at selected bed slopes and different discharges for gravel bed conditions (a) Smooth bed (b) Bed with 10 mm gravel size (c) Bed with 20 mm gravel size (d) Bed with 20 mm gravel size

 Table: 1

 Manning's roughness coefficient for different bed conditions for 0.25% slope

Discharge (q)	Bed slope (S) (%)	Manning's roughness coefficient (n)			
$(m^{3}s^{-1}m^{-1})$		Smooth bed	Bed with 10 mm gravel	Bed with 20 mm gravel	Bed with 30 mm gravel
0.088	0.25	0.00865	0.00994	0.01185	0.01382
0.053	0.25	0.00896	0.01115	0.01365	0.01661
0.038	0.25	0.00970	0.01214	0.01621	0.01971

increased, the depth flow decreased giving corresponding increase in flow velocity. However, with decrease in flow depth for a particular discharge the value of Manning's roughness coefficient increased for all selected discharges as shown in the Table 1 for the 0.25% slope.

#### Developed Mathematical Relationship for Manning's Roughness Coefficient in Terms of Bed Slope, Discharge and Gravel Size

In the present study, the Manning's roughness coefficient was observed for varying combinations of gravel size (D), bed slope (S) and discharge (q). Based on the observations, a mathematical relationship (MLR) was developed for Manning's roughness coefficient in terms of gravel size, bed slope and discharge.

MLR takes a group of random variables and tries to find a mathematical relationship among them. The model creates a relationship in the form of a straight line (linear) that best approximates all the individual data points of the study. The coefficient of determination of model is the quotient of the variances of the fitted values and observed values of the dependent variable. In general, multiple regression is to learn more about the relationship between several independent or predictor variables and a dependent or criterion variable. Once this so-called regression line has been determined, graph of the expected (predicted) and the observed values can be easily constructed. Using the techniques of MLR based on observed data, the following relationship was obtained for Manning's roughness coefficient. The mathematical relationship shown by eq. 9 is similar for this condition also as it is found in terms of discharge and bed slope.

## $n{=}0.012134{-}0.10242{*}q{+}0.000414{*}D{+}0.411349{*}S\\ \dots (9)$

This relationship is valid for the range of selected value of bed slope, discharge and gravel size. The calculated values of statistical performance indices like integral square

 Table: 2

 Calculated values of statistical performance indices

Parameters	Calculated value	
$\overline{\text{Coefficient of determination } (R^2)}$	0.8626	
Integral square error (ISE)	0.0155	
Coefficient of efficiency (CE)	0.8623	
Index of agreement (d)	0.9618	

error (ISE), coefficient of determination ( $\mathbb{R}^2$ ), coefficient of efficiency (CE) and index of agreement (d) were calculated for the observed and predicted values which are tabulated below in Table 2. As indicated in the Table 2, there was good agreement between the observed and predicted values as coefficient of determination was 0.8626 which shows a quite good agreement as shown in Table 2. Similarly, all other statistical indicators were also in satisfactory ranges such as ISE (0.0155), CE (0.8623) as well as Index of agreement (0.9618) as shown in Table 2.

#### 4. CONCLUSIONS

The experimental study on the effect of different sizes of gravel bed material roughness presented in terms of Manning's coefficient for different flow and slope conditions led to the following conclusions:

- The value of Manning's roughness coefficient was found to be increasing with the increasing size of bed material for every combination of bed slope and discharge.
- The value of Manning's roughness coefficient 'n' was found to be decreasing with increase in discharge for every combination of bed material and slope. However, the rate of decrease was rapid for lower bed slopes.
- The value of Manning's roughness coefficient was found to be increasing with increasing bed slope for every combination of bed material and discharge. It was however, observed that the value of Mannings roughness coefficient was increasing at a faster rate for lower values of discharge.
- A purpose of the study was to propose the appropriate equation for different slope, discharge with coarse-bed material. The overall mathematical relationship for Manning's roughness coefficient in terms of discharge (m<sup>3</sup>s<sup>-1</sup>m<sup>-1</sup>), size of bed material (mm) and bed slope (%) was obtained with satisfactory goodness of hit.
- On the basis of above discussion or results, it can be concluded that the developed relationship is capable of providing satisfactory estimate of Manning's roughness coefficient and was therefore used in further study for

relating energy dissipation and sequent depth ratio with the value of Manning's roughness coefficient for different combinations of input variable.

#### ACKNOWLEDGEMENTS

The authors are highly indebted to the Soil and Water Conservation Engineering Department, Govind Ballabh Pant University of Agriculture and Technology, Pantnagar for providing materials for this investigation and also express their sincere thanks to the Govind Ballabh Pant University of Agriculture and Technology, Pantnagar for providing necessary facilities during the study.

#### REFERENCES

- Barnes, H.H. 1967. Roughness characteristics of natural channels. U.S. Geological Survey Water Supply Paper 1849, 213p.
- Bray, D.I. 1979. Estimating Average Velocity in Gravel-Bed Rivers. J. Hydrau. Division, 105(9): 1103-1122.
- Chow, VenTe. 1959. Open-Channel Hydraulics New York, Mcgraw-Hill Book Co.
- Cowan, W.L. 1956. Estimating Hydraulic Roughness Coefficients. *Agric. Eng.*, 37(7): 473-475.
- Diskin, M.H., Ince, M. and Kwabena, O.N. 1978. Parallel cascades model for urban watersheds. Proceedings of American Society of Civil Engineers. J. Hydrau. Division, 104(2): 261-276.
- Jarret, R.D. 1985. Determination of roughness coefficients for streams in colorado. USGS Water Resources Investigation Report.
- Tzelepis, V., Moutsopoulos, K.N., Papaspyros, J.N. and Tsihrintzis, V.A. 2015. Experimental investigation of flow behavior in smooth and rough artificial fractures. J. Hydrol., 521:108-118.
- Legates, D.R. and McCabe, G.J. 1999. Evaluating the use of "goodness of fit" measures in hydrologic and hydro-climatic model validation. *Water Resour. Res.*, 35(1): 233-241.
- Limerinos, J.T. 1970. Determination of Manning coefficient for measured bed roughness in alluvial channels. Water Supply Paper 1898-B, United States Geological Society, Washington, D.C.
- López, R. and Barragán, J. 2008. Equivalent Roughness of Gravel-Bed Rivers. J. Hydrau. Eng., 134(6): 847-851.
- Mishra, P.K., Ramajayam, D., Sankar, G.R. and Math, S.K.N. 2018. Effect of runoff and soil loss on soil hydraulic parameters and forage yield of four grass species under arid vertisols. *Indian J. Soil Cons.*, 46(1): 34-41.
- Meyer-Peter, E. and Muller, R. 1948. Formulas for Bed-Load Transport. *Proc. of the 3<sup>rd</sup> Meeting of Iahr, 39-64.* Stockolm: International Association for Hydraulic Research.
- Nash, J.E. and Sutcliffe, J.V. 1970. River flow forecasting through conceptual models part I – A discussion of principles. J. Hydrol., 10(3): 282-290.
- Shamseldin, A., Y., O'Conner, K.M. and Liang, G.C. 1997. Methods for combining the outputs of the different rainfall – runoff models. J. Hydrol., 197: 203-229.
- Strickler, A. 1923. Roesgan, T. and Brownie, W.R., Trans. 1981. Contributions to the Question of A Velocity Formula and Roughness Data for Streams, Channels and Closed Pipelines. Pasadena: W.M. Keck Lab of Hydraulics and Water Resources, California Institute of Technology.
- Willmott, C.J. 1981. On the validation of models. Phys. Geogr., 2: 184-194.
- Zeng, C., Li, C., Tang, H., Wang, L. and Mao, J. 2015. Experimental study of depth-limited open-channel flows over a gravel bed. *Int. J. Sedim. Res.*, 30(2): 160-166.